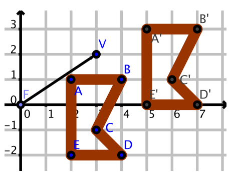
Coordinate transformations in a nutshell

In algebra, you learned about *functions*, which are rules that take an input number and produce an output number. For example, the function $f(x) = x + 3$ takes and input number and outputs a number that is bigger by 3. You can also think of functions geometrically by considering them as *transformations* of *points* on a number line. In this case, you can think of f as taking its input and "moving" it 3 to the right. This particular kind of transformation, which moves all input points the same distance in the same direction, is called a *translation*. (The words *translation* and *transformation* sound similar but are different: a translation is a kind of transformation.)

In geometry we often extend the transformation idea to points of two, three, or even more dimensions. We'll confine ourselves to two dimensions here. In two dimensions, a transformation looks algebraically like a function: $f(x_i, y_i) = (x_o, y_o)$ (Where (x_i, y_i) is the input point and (x_o, y_o) is the output point.)

Because a transformation operates on points, we can ask what a transformation does to a whole geometric figure by asking what it does to each of its points. Just like in one dimension, a two-dimensional *translation* moves all input points the same distance in the same direction. Here is an example of a polygon $ABCDE$ and the result of its translation by $f(x, y) = (x + 3, y + 2)$.



Transformations play a key role in computer graphics, medical imaging technologies like CT scans, and cartography. They also are also likely to play a minor role in our (ahem) final.

1. Now define translations that:

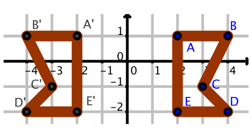
   a) Move any point 3 to the right

   b) Move any point down 2

   c) Move any point "northeast" (same distance right and up) a distance of $\sqrt{2}$

   d) Is the translated figure congruent, similar, or neither to the original figure?

2. Another kind of transformation is a *reflection*. A *reflection* transforms any point onto its mirror image about a line called the *axis of symmetry*. Here's a *reflection about the y-axis*:

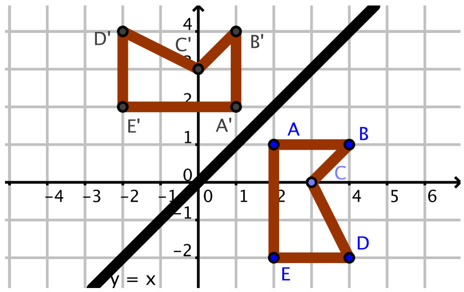
   a) Define the above reflection algebraically: $f(x, y) =$

   b) Define, algebraically, a reflection about the x-axis: $f(x, y) =$

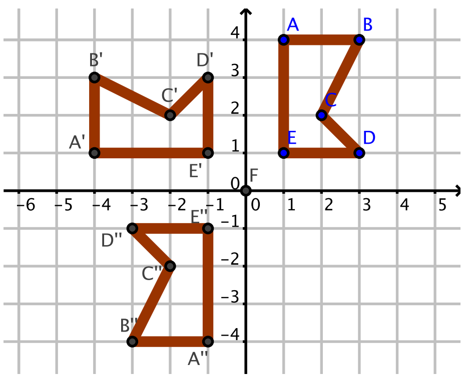
   c) Is the translated figure congruent, similar, or neither to the original figure?

   d) Define, algebraically, a reflection about the line $y = x$. $f(x, y) =$

   (Try a few points on the graph below and see if you see the pattern. By the way, If you use this reflection on the graph of any invertible mathematical function, the reflected graph is the graph of the inverse function!)



3. Yet another kind of transformation is a *rotation*. Here is a figure rotated 90° counterclockwise, and then the rotated figure is rotated again using the same transformation.

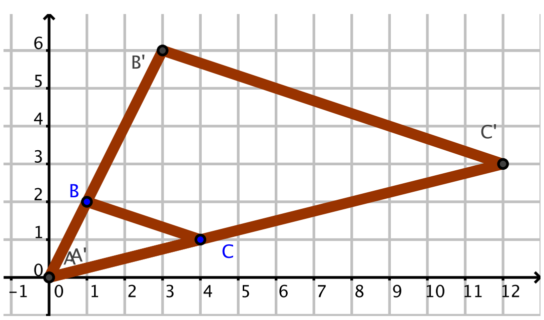


a) Define this function algebraically: $f(x,y) =$

(Again, look at how a few points from the original figure get transformed, say $A \to A'$ and $B \to B'$. Once you think you have it, see if the same mapping works to map $A' \to A''$ etc.)

b) Is the translated figure congruent, similar, or neither to the original figure?

4. Dilations are transformations that enlarge or shrink a figure proportionally. Here is a triangle ABC and the result of a dilation $A'B'C'$. (In this case, $A = A'$.)



a) Define this function algebraically: $f(x,y) =$

b) Is the translated figure congruent, similar, or neither to the original figure?

c) What is the slope of $\overline{BC}$? Of $\overline{B'C'}$