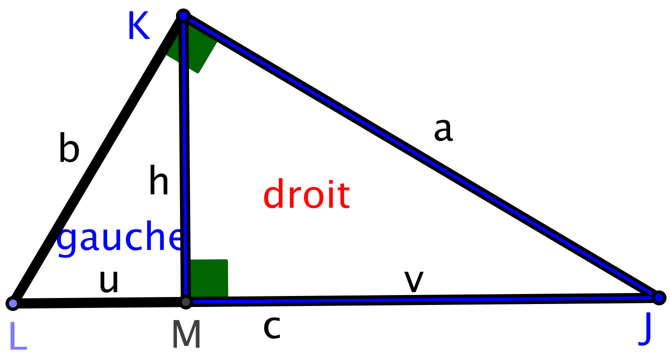
In this worksheet, you will write an informal proof of the Pythagorean Theorem.



This entire worksheet refers to the above picture. Right off the bat, notice the labels a, b and c which represent the legs and hypotenuse of the big right triangle, which should remind you of the Pythagorean Theorem, which tells us, in a right triangle with legs a and b and hypotenuse c, that $a^2 + b^2 = c^2$. So you know the punch line already. Keep your eyes on that prize as you do this worksheet.

Because it's easy to get caught up in alphabet soup, we'll sometimes refer to $\triangle KML$ as $\triangle gauche$ (French for "left"), $\triangle JMK$ as $\triangle droit$ ("right"), and $\triangle JKL$ as $\triangle grand$ ("big").

There's a slight ambiguity in the labeling of the segments on the bottom. In this picture, u is the length of the short leg of $\triangle gauche$; v is the length of the long leg of $\triangle droit$; and c is the length of the hypotenuse of $\triangle grand$. Notice that $u + v = c$.

Whenever I ask "why," write a reason you could use in a proof. Sometimes a one-word answer is enough.

1. First we need to remind ourselves why $\triangle gauche \sim \triangle grand$. You know that $\angle L \cong \angle L$ because of the reflexive property. Why is $\angle KML \cong \angle JKL$?

2. Why is $\triangle gauche \sim \triangle grand$?

3. Let's use the same technique on $\triangle droit$. You know that $\angle J \cong \angle J$ because of the reflexive property. Why is $\angle JMK \cong \angle JKL$?

4. Why is $\triangle droit \sim \triangle grand$?

5. Ok, now we have three similar triangles. Hold on to that. Write the ratio of the short leg of $\triangle gauche$ to the short leg of $\triangle grand$.

6. Write the ratio of the hypotenuse of $\triangle gauche$ to the hypotenuse of $\triangle grand$.

7. Because $\triangle gauche \sim \triangle grand$, what can you say about those ratios? Write that as an equation.

8. Cross-multiply that equation to get an equivalent equation without fractions and label that *Equation 1.* For consistency, write the product of two different values on the left and the square on the right. (You *did* get a product of two different values on one side and the square on the other, right?)

   By the way, we could have jumped directly to this point using the Altitude-On-Hypotenuse Theorem.

   Equation 1.

9. Now let's work on what will become *Equation 2*. Write the ratio of the long leg of $\triangle droit$ to the long leg of $\triangle grand$.

10. Write the ratio of the hypotenuse of $\triangle droit$ to the hypotenuse of $\triangle grand$.

11. Because $\triangle droit \sim \triangle grand$, what can you say about those ratios? Write that as an equation.

12. Cross-multiply that equation to get an equivalent equation without fractions and label that *Equation 2*. For consistency, write the product of two different values on the left and the square on the right.

    We also could have jumped directly to this point using the Altitude-On-Hypotenuse Theorem.

    Equation 2.

13. Now we have two equations. Since we have shown equations 1 and 2 to be true, using basic algebra we know that the sum of the left sides also must equal the sum of the right sides. Write that new equation and call it (yes I'm really this creative) *Equation 3*.

    Equation 3.

14. Use your friend, the Distributive Law ($xz + yz = (x + y)z$) to simplify the left side. Write that new equation and call it *Equation 4*. Do you smell the barn yet?

    Equation 4.

15. Ok, now take a look at the picture again. Do you see a simpler name for the sum in *Equation 4*? Rewrite the equation and label that the *Pythagorean Theorem*. You rock!

    Pythagorean Theorem