

## Level 1 Geometry Review Topics

- Logic – If-then, inverse, converse, and contrapositive
- Complementary and Supplementary Angles
- Vertical Angles
- Congruent Triangles – ASA, SAS, SSS, AAS, HL, CPCTC
- Altitudes, Medians, and Angle Bisectors of triangles
- Types of triangles – Right, Obtuse, Acute, Scalene, Isosceles, Equilateral
- Isosceles Triangles – Base Angles Theorem
- Exterior Angles
- Coordinate Geometry – Slope, Distance Formula, Midpoint Formula
- Parallel Lines – Alternate Interior Angles, Corresponding Angles, Same-side Interior, Alternate Exterior Angles, Same-side Exterior Angles.
- Quadrilaterals – parallelogram, rectangle, rhombus, square, kite, trapezoid
- Properties of quadrilaterals
- Sum of interior angles of a triangle =  $180^\circ$
- Sum of exterior angles of any polygon =  $360^\circ$
- Sum of interior angles of any polygon
- No Choice Theorem
- Names of polygons – pentagon, hexagon, etc.
- Regular Polygons
- Proportions
- Similar Triangles – AA Similarity, SAS Similarity, SSS Similarity
- Side-splitter theorem
- Midline Theorem
- Geometric Mean with Right Triangles
- Pythagorean Theorem
- Pythagorean Triples
- Special Right Triangles – 30-60-90, 45-45-90
- Pythagorean Theorem in Space Figures
- Trigonometry – Sine, Cosine, and Tangent
- Circles – central angles, inscribed angles, secants, tangents, chords
- Areas and Volumes
- Transformations – reflection, translation, rotation, and dilation
- Tessellations

## Level 1 Geometry Review 1: Angles and Polygons

Polygon angle formulas - Let  $n$  = the number of sides in a polygon.

The sum of the interior angles in any  $n$ -gon:  $(n - 2) 180^\circ$

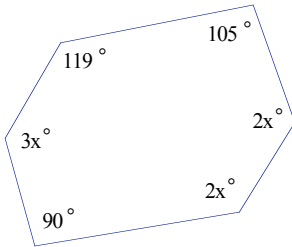
One interior angle in a regular  $n$ -gon:  $\frac{(n - 2)180^\circ}{n}$

The sum of the exterior angles in any  $n$ -gon:  $360^\circ$

The measure of an exterior angle in a regular  $n$ -gon:  $\frac{360^\circ}{n}$

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1. Define a regular polygon.
2. What is the sum of the interior angles of a 40 sided polygon?
3. Given the angles shown in this polygon, find the value of  $x$ .



4. Given a regular 90-gon how large is each exterior angle?
5. What is the size of each interior angle of a regular dodecagon (12 sided polygon)?
6. Suppose a regular polygon has exterior angles of size  $0.5^\circ$ . How many sides are there?
7. Suppose a regular polygon has each interior angle of size  $170^\circ$ . How many sides are there?
8. Suppose a polygon's interior angles add to  $7200^\circ$ . How many sides are there?

9. True or false? A polygon's interior angles can add up to  $1970^\circ$ .
10. In a regular polygon each interior angle is  $170^\circ$  larger than each exterior angle.  
How many sides are there?
11. The supplement of an angle is 6.5 times as large as the angle. Find the angle.
12. The angles of a triangle are in the ratio of 5:5:6. Find the angles.
13. The complement of an angle is  $22^\circ$  more than the angle. Find the angle.
14. Twice the supplement of an angle is  $230^\circ$ . Find the angle.
15. Half the complement of an angle when added to one quarter of the angle makes  $32.5^\circ$ .  
Find the angle and its complement.
16. Two sides of a triangle are of lengths 5 and 14. What are all the possible lengths for the third side?

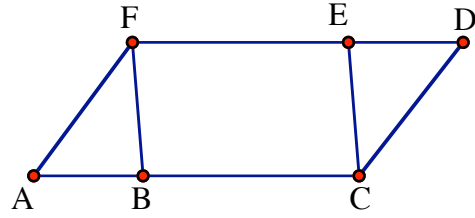
## Level 1 Geometry Review 2: Congruent Triangles

1. Vertical angles are congruent.
2. Base Angles Theorem – If 2 sides of a triangle are congruent, then the opposite angles are congruent. And its converse – If 2 angles of a triangle are congruent then the opposite sides are congruent.
3. Proving triangles congruent: ASA, SAS, SSS, AAS, and HL
4. **Properties of parallelograms**
  - Opposite sides are parallel
  - Opposite sides are congruent
  - Opposite angles are congruent
  - Diagonal bisect each other
5. **Properties of rectangles**
  - All properties of a parallelogram
  - All angles are right.
  - Diagonals are congruent.
6. **Properties of a rhombus**
  - All properties of a parallelogram
  - All sides are congruent.
  - Diagonals are perpendicular.
  - Diagonals bisect opposite angles.
7. **Properties of a square**
  - All properties of a rectangle
  - All properties of a rhombus
8. **Properties of an isosceles trapezoid**
  - One pair of opposite sides parallel
  - Legs are congruent.
  - Base angles are congruent.
  - Diagonals are congruent.
9. Besides the properties, you can prove that a quadrilateral is a parallelogram by proving that one pair of opposite sides are both congruent and parallel.

Problems.

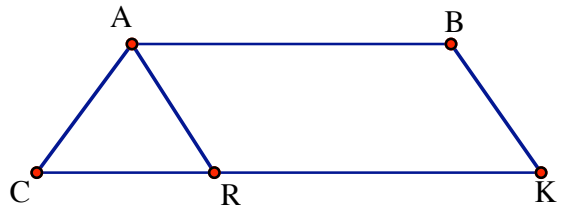
1. Given:  $ACDF$  is a parallelogram.  
 $\angle AFB = \angle ECD$

Prove:  $FBCE$  is a parallelogram.



2. Given:  $\overline{AC} \cong \overline{AR}$   
 $\overline{AC} \cong \overline{BK}$   
 $\angle C \cong \angle K$

Prove:  $BARK$  is a parallelogram.

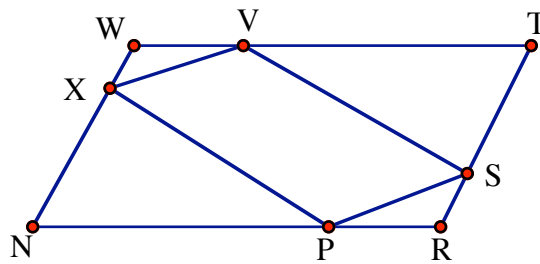


3. Given:  $NRTW$  is a parallelogram.

$$\overline{NX} \cong \overline{TS}$$

$$\overline{WV} \cong \overline{PR}$$

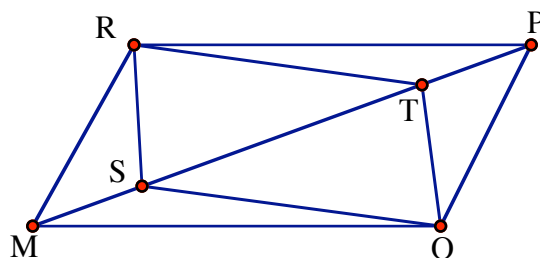
Prove:  $XPSV$  is a parallelogram.



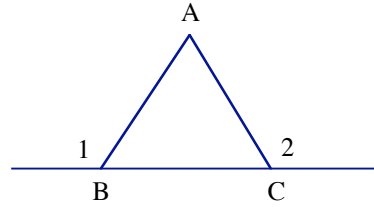
4. Given:  $RSOT$  is a parallelogram.

$$\overline{MS} \cong \overline{TP}$$

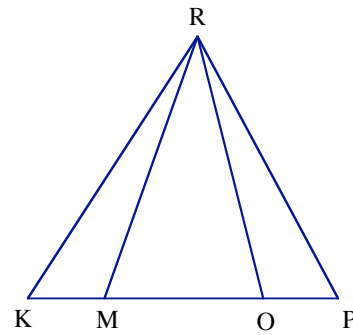
Prove:  $MOPR$  is a parallelogram.



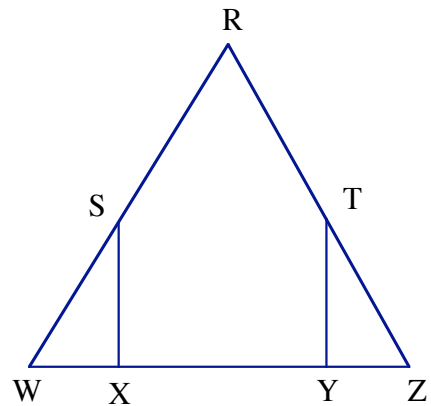
5. Given:  $\overline{AB} \cong \overline{AC}$   
Prove:  $\angle 1 \cong \angle 2$



6. Given:  $\angle KRM \cong \angle PRO$   
 $\overline{KR} \cong \overline{PR}$   
Prove:  $\overline{RM} \cong \overline{RO}$



7. Given:  $\overline{SX} \cong \overline{TY}$   
 $\overline{WX} \cong \overline{YZ}$   
 $\overline{SW} \cong \overline{TZ}$   
Prove:  $\overline{RW} \cong \overline{RZ}$



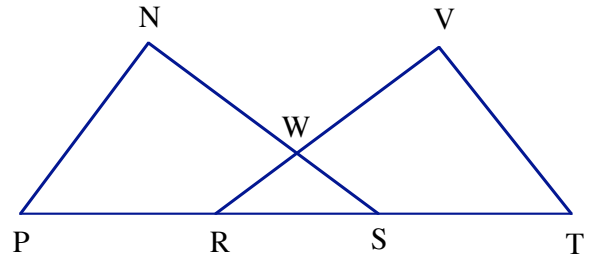
8.

$$\overline{PR} \cong \overline{ST}$$

Given:  $\overline{NP} \cong \overline{VT}$

$$\angle P \cong \angle T$$

Prove:  $\triangle WRS$  is isosceles.

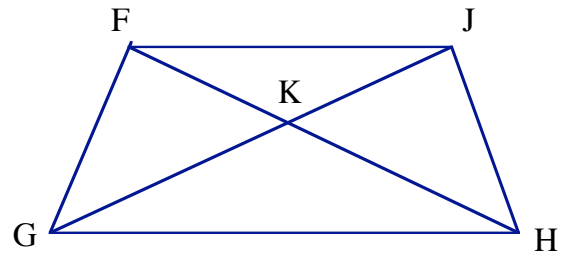


9.

Given:  $\overline{FG} \cong \overline{JH}$

$$\angle FGH \cong \angle JHG$$

Prove:  $\triangle FKJ$  is isosceles.

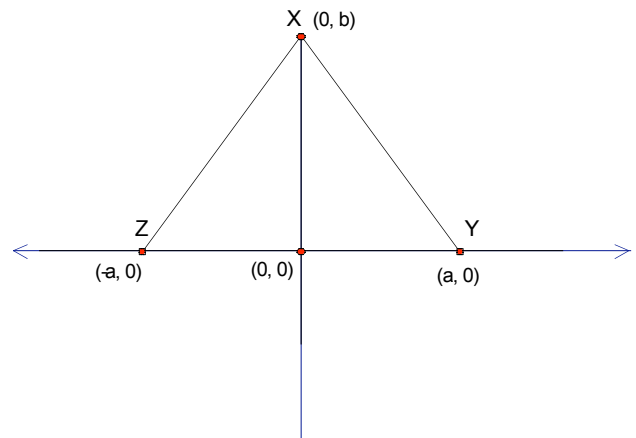


### Level 1 Geometry Review 3

1. A coordinate proof.

Given: any isosceles triangle, positioned cleverly as shown.

Prove that the medians of  $\triangle XYZ$  from Y and Z are congruent.



Reminders:

- A median is a line segment from a vertex of a triangle to the midpoint of the other side.

- Midpoint formula:

$$\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- Distance formula:

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

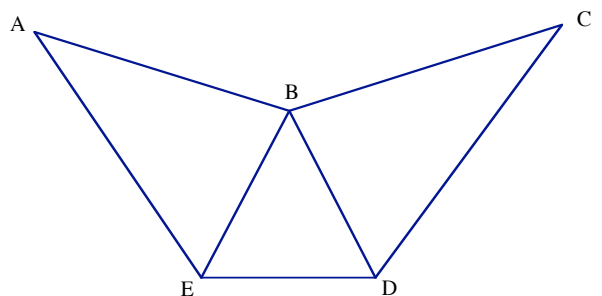
2. Given:

$$\angle BED \cong \angle BDE$$

$$\angle BEA \cong \angle BDC$$

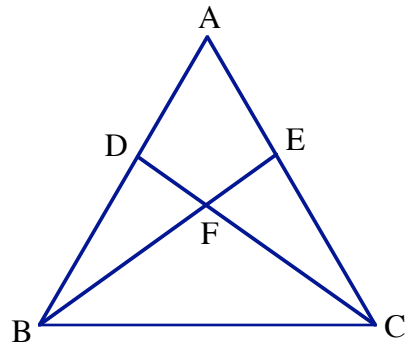
$$\overline{AE} \cong \overline{CD}$$

Prove:  $\angle A \cong \angle C$



3. Given:  $\overline{AB} \cong \overline{AC}$   
 $\overline{AD} \cong \overline{AE}$

Prove  $\triangle FBC$  is isosceles

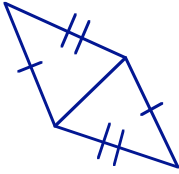


## Level 1 Geometry Review 4

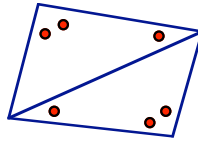
The following are valid reasons for proving that two triangles are congruent:  
 ASA, AAS, SAS, SSS, HL. Other reasons are not enough: AAA, SSA

1. In each diagram state why the triangles are congruent or write “NN” (not necessarily congruent).  
 Pay no attention to the appearance of the triangles, only the facts given.

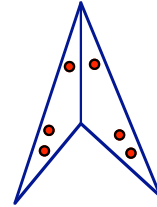
a.



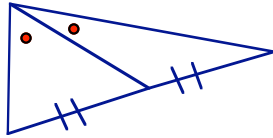
b.



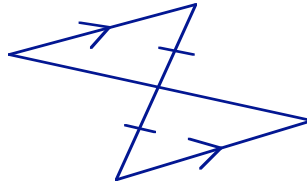
c.



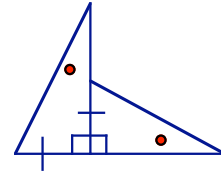
d.



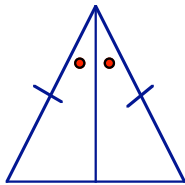
e.



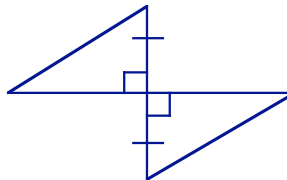
f.



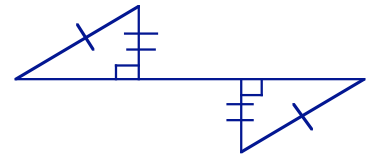
g.



h.



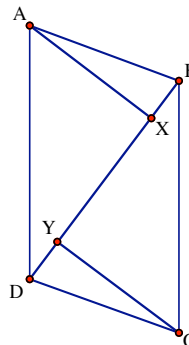
i.



2.

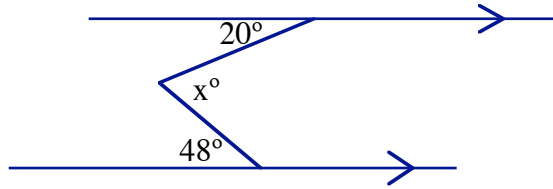
Given:  $AB = CD$ .  
 $AX \perp BD$ .  
 $CY \perp BD$ .  
 $BX = DY$ .

Prove: ABCD is a parallelogram.

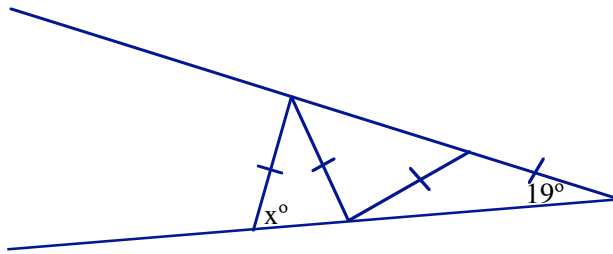


3. Find the value of  $x$ .

a.



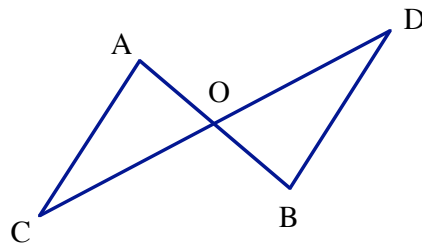
b.



4.

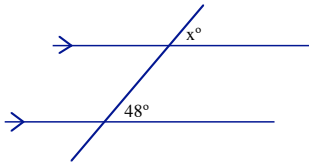
Given:  $\overline{AO} \cong \overline{OB}$   
 $\overline{AC} \parallel \overline{BD}$

Prove:  $\triangle AOC \cong \triangle BOD$

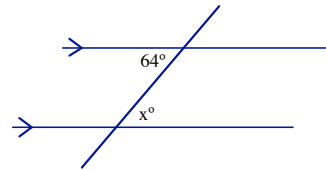


5. Find  $x$ .

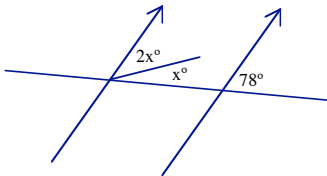
a.



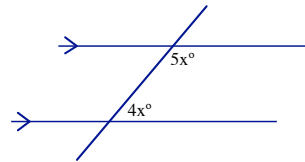
b.



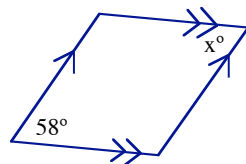
c.



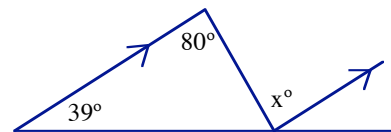
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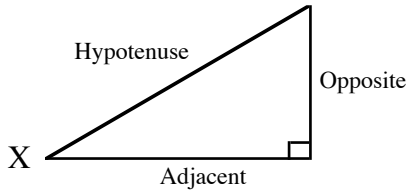
e.



f.



## Level 1 Geometry Review 5



$$s = \frac{o}{h} \quad c = \frac{a}{h} \quad t = \frac{o}{a}$$

$$\sin(x) = \frac{\textit{opposite}}{\textit{hypotenuse}}$$

$$\cos(x) = \frac{\textit{adjacent}}{\textit{hypotenuse}}$$

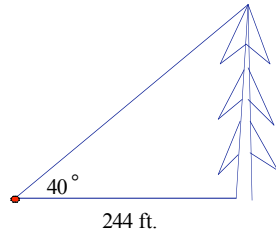
$$\tan(x) = \frac{\textit{opposite}}{\textit{adjacent}}$$

Examples:

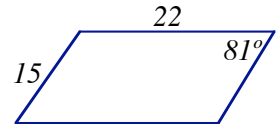
	$\cos(18^\circ) = \frac{140}{x}$ $x \cos(18^\circ) = 140$ $x = \frac{140}{\cos(18^\circ)}$ $x = 147.2$
	$\tan(x) = \frac{130}{275}$ $x = \tan^{-1}\left(\frac{130}{275}\right)$ $x = 25.3^\circ$

Problems:

1. How high is the tree?

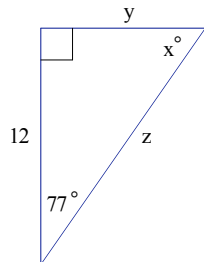


2. Find the height of this parallelogram and then its area.



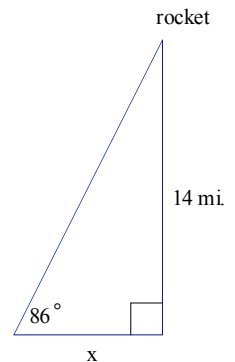
height =  
area =

3. Find all missing sides and angles.  
*Note that the third side can be found by the Pythagorean Theorem or trig.*



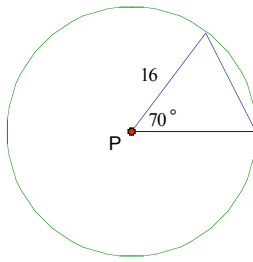
x =  
y =  
z =

4. While watching a spaceship take off you hear on the radio that it is 14 miles in the air. You have to look up at an angle of  $86^\circ$  to see it. How far from the blastoff position are you?

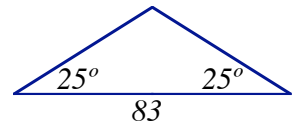


5. Given  $\odot P$ , find the area of the shaded segment.

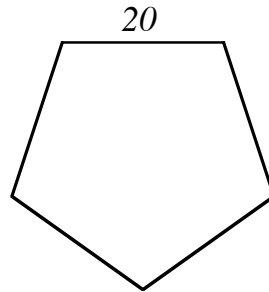
Area of sector =  
 Area of triangle =  
 Area of segment =



6. Find the perimeter of the triangle.



7. Find the area of this regular pentagon by dividing it into triangles from the center.

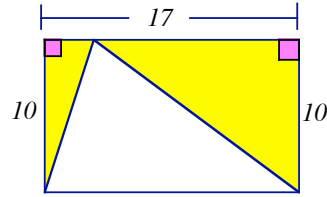


8. Without a calculator, you should be able to find these values by drawing and labeling familiar triangles.
- $\tan(45^\circ) =$   
 $\cos(60^\circ) =$   
 $\sin(45^\circ) =$

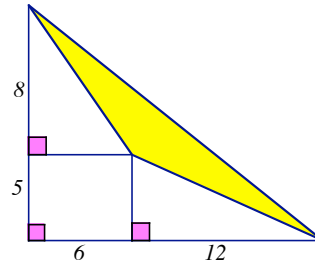
## Level 1 Geometry Review 6

### Area, Surface Area, Volume

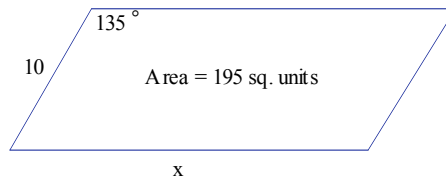
1. Find the shaded area.



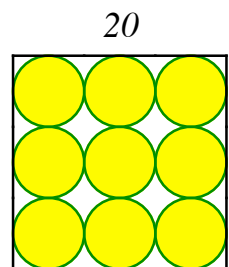
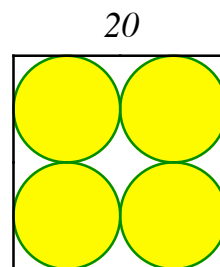
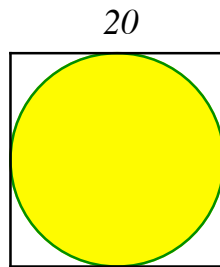
2. Find the shaded triangular area.



3. This figure is a parallelogram. Find length  $x$ .

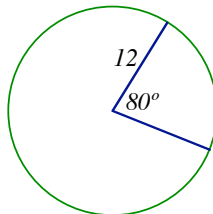


4. In each figure congruent circles just fit in congruent squares with sides of length 20. For each one find the percentage of the square's area occupied by the circles.



Explain your result.

5. Find the sector area.



6. This is a semicircle. Its circumference is 65. What is its radius?

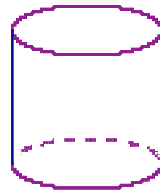


7. A cylinder has a base area of  $25\pi$  square inches and a volume of  $375\pi$  cubic inches.

Base radius =

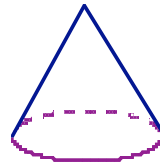
Altitude =

Lateral area =



8. A cone has a volume of 1500 cubic inches. The altitude is equal to the diameter of the base.

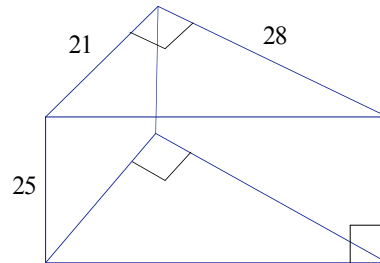
Altitude =



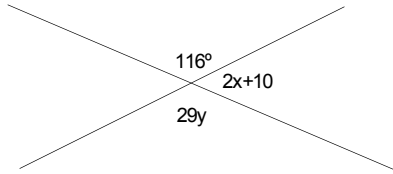
9. A right prism has a right triangular base. The legs of the base are 21 and 28 and the altitude of the prism is 25.

Volume =

Total surface area =



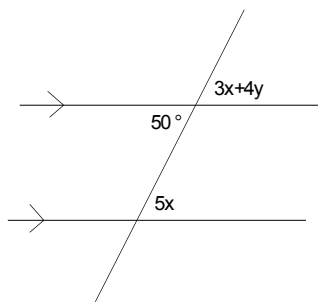
10.



$x =$

$y =$

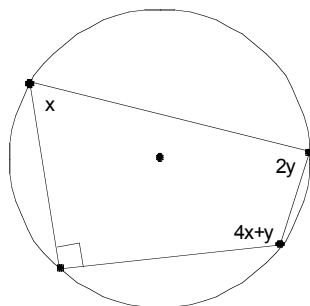
11.



$x =$

$y =$

12.



$x =$

$y =$

## Level 1 Geometry Review 7

### Conditional or If-Then statements.

Suppose a certain statement "If P then Q" is true.

Original statement:	If p then q. ( $p \Rightarrow q$ )	Suppose this is true.
Converse:	If q then p. ( $q \Rightarrow p$ )	This is not necessarily true.
Inverse:	If not p then not q. ( $\sim p \Rightarrow \sim q$ )	This is not necessarily true.
Contrapositive:	If not q then not p. ( $\sim q \Rightarrow \sim p$ )	Always true if the original statement is true.

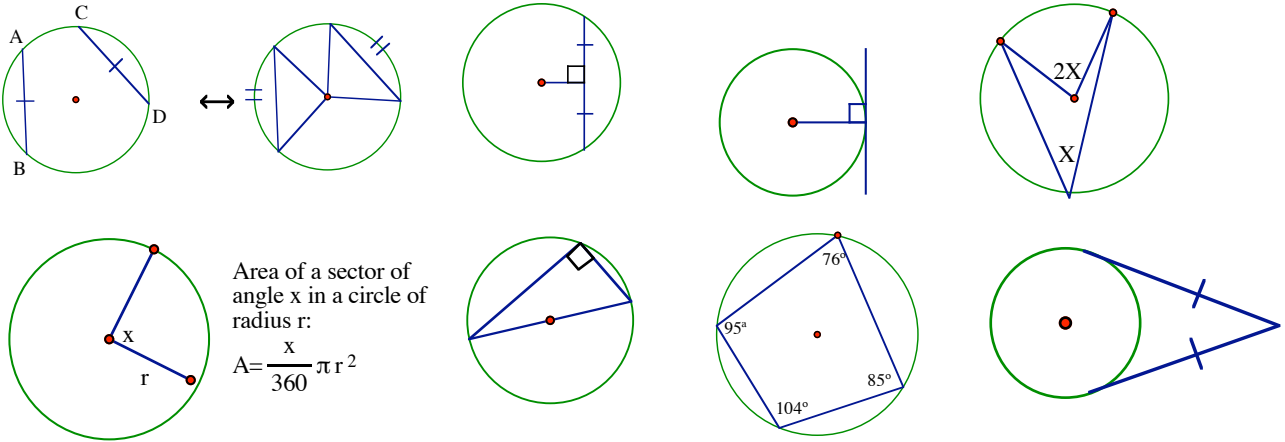
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1. Assume this statement is always true:  
If you bang your head against the wall you will get a headache.
  - a. Make a Venn diagram for this situation.
  - b. Write the converse and state whether it is necessarily true.
  - c. Write the inverse and state whether it is necessarily true.
  - d. Write the contrapositive and state whether it is necessarily true.
  - e. Joe Max banged his head against the wall. Put Joe Max in the diagram (possibly in more than one place) and draw any conclusion you can about him.
  - f. Ozzie has never had a headache. Put Ozzie in the picture, wherever possible, and draw any conclusion you can.
  - g. Latrice has a headache. Place her and draw a conclusion if you can.
  - h. Charmaine has never banged her head against the wall. Place her and conclude what you can.
  
2. Assume the following to be always true statements:  
A watched pot of water never boils.  
To be able to cook spaghetti you must boil a pot of water.

Determine whether the following are always true, always false or neither.

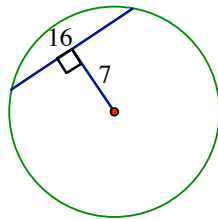
- a. If you are cooking spaghetti you did not watch the pot.
- b. If you don't watch a pot of water, it will boil.
- c. If you boil water you can cook some spaghetti.
- d. If you are boiling water you did not watch the pot.

**Circle theorems:**

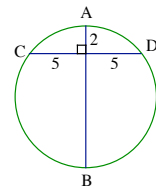


Area of a sector of angle  $x$  in a circle of radius  $r$ :  
 $A = \frac{x}{360} \pi r^2$

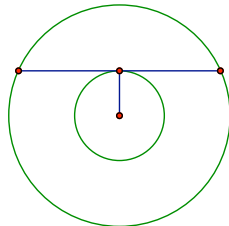
**3.** Find the radius of the circle.



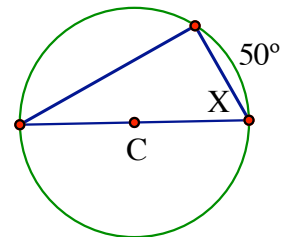
**4.** Find the radius.



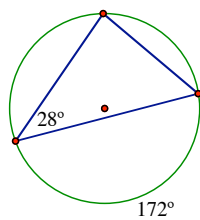
**5.** Two concentric circles have radii of 17 and 7. The chord is tangent to the small circle. What is its length?



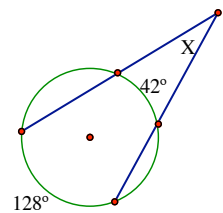
**6.** C is the center of this circle and the triangle is inscribed in it. Find the size of angle X.



**7.** Find the missing angles and arcs.



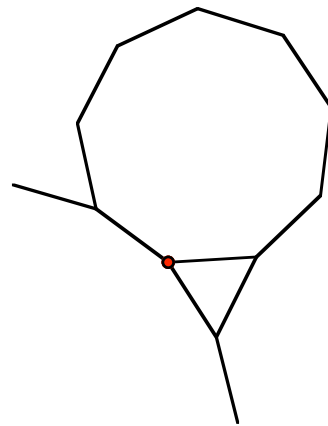
**8.** Find angle X.



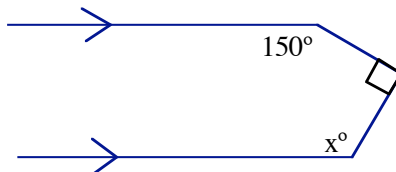
## Level 1 Geometry Review 8

1. An architect is designing a house and builds a scale model. The model is 2 feet high and the house will be 24 feet high. How do the heights of the front doors compare, the floor areas of the master bedrooms compare, and the volume of the attics compare in the model to the actual house?

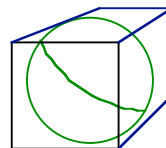
2. Three regular polygons surround a point. How many sides has the largest polygon?



3. Find  $x$ .



4. A sphere just fits in a cube whose sides are of length 10. What percentage of the cube's volume is occupied by the sphere?



## Level 1 Geometry Review 9

### 1. Coordinate transformations.

Be prepared to recognize TRANSLATIONS, ROTATIONS, REFLECTIONS and combinations like GLIDE REFLECTIONS.

For each problem:

i. Transform each figure according to the transformations given by applying them to the vertices of the figure and then connecting the new vertices. Draw the new figure on the same graph.

Here's the notation:  $x_1$  is the starting  $x$ -coordinate of a point.

$x_2$  is the  $x$ -coordinate of the point after the transformation.

$y_1$  is the starting  $y$ -coordinate of a point.

$y_2$  is the  $y$ -coordinate of the point after the transformation.

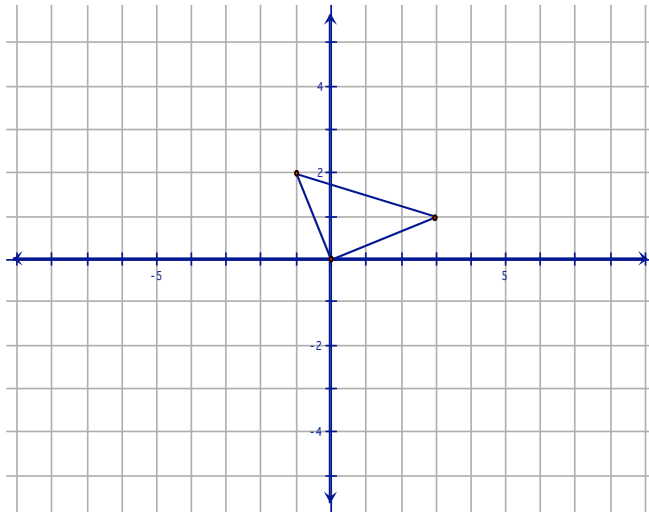
ii. Describe the transformation in words using the terminology above, if it applies.

FOR TRANSLATIONS SPECIFY HOW FAR AND IN WHAT DIRECTION.

FOR ROTATIONS SPECIFY THE CENTER AND ANGLE TURNED.

FOR REFLECTIONS SPECIFY THE LINE OF REFLECTION.

a.

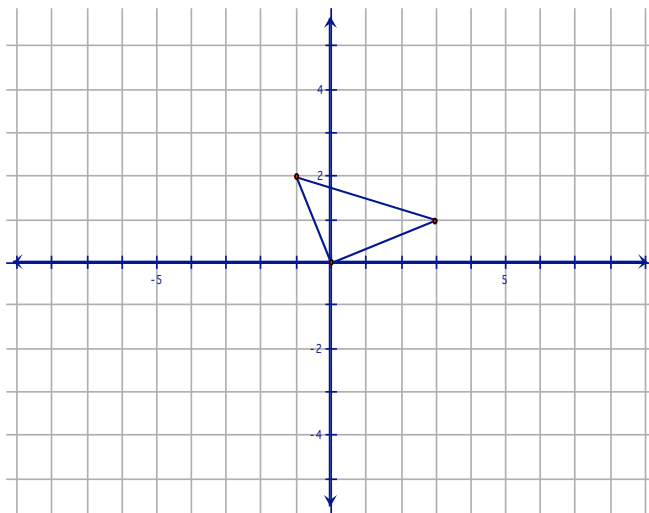


$$x_2 = x_1 - 5$$

$$y_2 = y_1 + 3$$

Describe the transformation.

b.

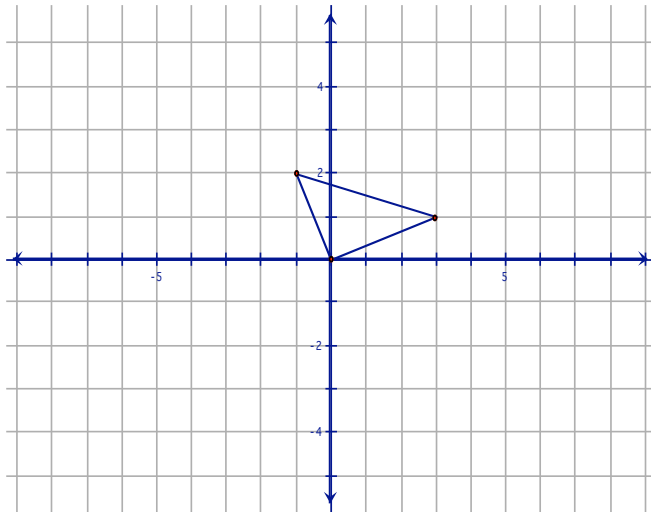


$$x_2 = x_1$$

$$y_2 = -2 - y_1$$

Describe the transformation.

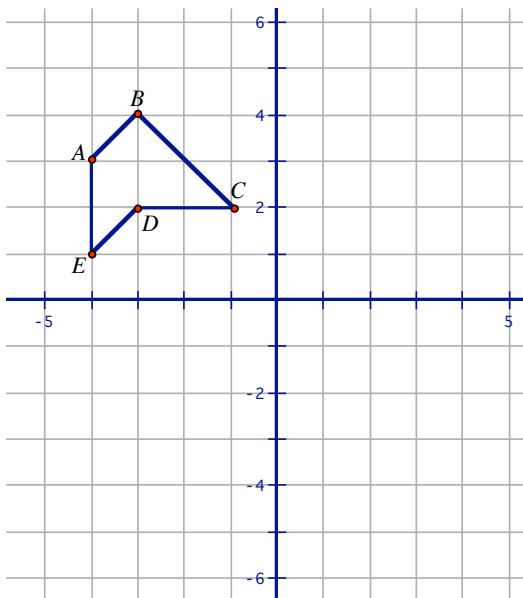
c.



$$x_2 = \frac{1}{2}x_1$$
$$y_2 = 4y_1$$

Describe the transformation.

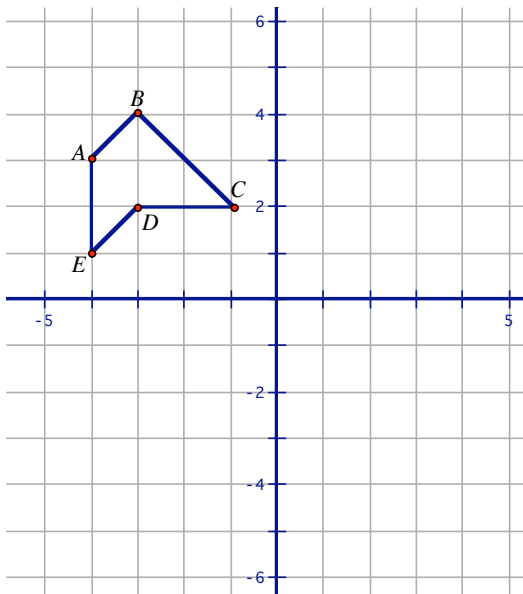
d.



$$x_2 = y_1$$
$$y_2 = x_1$$

Describe the transformation.

e.



$$x_2 = -x_1$$
$$y_2 = 6 - y_1$$

Describe the transformation.

Tools of analytic geometry. Given two points  $(x_1, y_1), (x_2, y_2)$ :

Distance formula:  $D = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Midpoint formula:  $M = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Slope formula:  $m = \frac{y_2 - y_1}{x_2 - x_1}$  Parallel lines have equal slopes.

Perpendicular lines have negative reciprocal slopes.

Quadratic formula.  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

1. Here are the vertices of a triangle: A(2, -6), B(12, 18), C(-10, -1).

Calculate the slopes of the three sides.

$m_{AB} =$                        $m_{BC} =$                        $m_{AC} =$

Why is the triangle a right triangle?

Which is the right angle?

Find the area of the triangle by any method you choose. Show your work.

2. Show that triangle PDQ is isosceles but not equilateral: P(1, 6), D(7, -1), Q(-1, -3).

3. A circle has a diameter AB with A(-1, 2) and B(23, 9).

a. Find the center of the circle.

Center: (     ,     )

b. Find the radius of the circle:

Radius: