

1 Matrices

Matrix: a rectangular array of numbers, arranged in rows and columns, like a spreadsheet, e.g.

$$\begin{pmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{pmatrix}$$

This matrix has two *rows* and three *columns*.

- *row matrix*: $(1 \ 2 \ 3)$
- *column matrix*: $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$
- *square matrix*: $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$
- *zero matrix*: $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- *corresponding entries*: entries in different matrices but the same row and column, e.g. $\begin{pmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{pmatrix} \begin{pmatrix} 11 & 12 & 13 \\ 21 & 22 & 23 \end{pmatrix}$
- Matrices are *equal* if their corresponding entries are equal: $(2+3 \ 4) = (5 \ 2^2)$
- Adding or subtracting matrices
 - Matrices must have same dimensions
 - Just add or subtract corresponding entries
 - $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} + \begin{pmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{pmatrix} = \begin{pmatrix} 1+2 & 2+3 & 3+4 \\ 4+5 & 5+6 & 6+7 \end{pmatrix} = \begin{pmatrix} 3 & 5 & 7 \\ 9 & 11 & 13 \end{pmatrix}$
- A *scalar* is just a number (as distinguished from a matrix)
- Scalar multiplication: multiply each entry by the scalar:
- $3 \cdot \begin{pmatrix} 0 & 1 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 3 \cdot 0 & 3 \cdot 1 \\ 3 \cdot (-2) & 3 \cdot 4 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -6 & 12 \end{pmatrix}$
- Algebraic properties follow from similar properties of scalars:
 - Associative property of addition: $(A + B) + C = A + (B + C)$
 - Commutative property of addition: $A + B = B + A$ (for example: $(1 \ 2) + (3 \ 4) = (1+3 \ 2+4) = (3+1 \ 4+2) = (3 \ 4) + (1 \ 2)$)
 - Distributive property (multiplication by a scalar): $c(A + B) = cA + cB$
- Example: MA voting (just the questions). Each precinct might have a 2-column, 3-row matrix where the first column represents “yes” votes and the 2nd column represents “no”

votes. $\begin{matrix} Q1 \\ Q2 \\ Q3 \end{matrix} \begin{pmatrix} 1000 & 800 \\ 900 & 700 \\ 600 & 500 \end{pmatrix}$