1 Solving linear systems algebraically: substitution and linear combination

2 Warmup

Try to solve this system algebraically or graphically. Remember that solving the system means finding a solution that works for all equations.

$$2\,x + 2\,y = 10$$

$$y = \frac{3}{2}x$$

3 Misc

Collect homework

Homework for next class: Homework: Read section 3.2, p.148-151. Problems: p.152-153 1-50 odd.

4 Hybrid car again

Remember the hybrid car from the test? We said it gets 40 miles/gallon and 2 miles/kw-h. Let's also estimate that a gallon costs \$3 and a kw-h costs \$.1. How would we find out how much of each kind of fuel to use if we wanted to go 200 miles and spend \$13?

5 Substitution

Logic: You can substitute equal things for each other without changing the solution. This method is especially convenient when a coefficient is 1.

5.1 Example

Solve:

$$\begin{aligned} 3\,x + 4\,y &= -4 \\ x + 2\,y &= 2 \end{aligned}$$

Solution: Solve one equation for x or y, making a revised equation:

$$\begin{aligned} x + 2\,y &= 2 \\ x &= -2\,y + 2 \end{aligned}$$

...and substitute in the other equation and solve for the remaining variable:

$$\begin{aligned} 3\,x + 4\,y &= -4 \\ 3\,(-2\,y + 2) + 4y &= -4 \\ -6\,y + 6 + 4\,y &= -4 \\ -2\,y &= -10 \\ y &= 5 \end{aligned}$$

...then plug that in and solve the revised equation for the first variable:

$$\begin{aligned} x &= -2y+2 \\ &= -2\cdot 5+2 \\ &= -8 \end{aligned}$$

So the solution is $(-8,5)$.

Check your work! Make sure you have a solution. For example, plug in to the first equation:

$$\begin{aligned} 3x+4y &= 3\cdot(-8)+4\cdot 5 \\ &= -24+20 \\ &= -4\checkmark \end{aligned}$$

Graph it to visualize.

6 Linear combination

6.1 Example

The simplest linear combination case is when the two equations add up to yield a new equation in only one variable:

$$\begin{aligned} 2x+y &= 3 \\ -3x-y &= -5 \end{aligned}$$

You can add them to get

$$\begin{aligned} -x &= -2 \\ x &= 2 \end{aligned}$$

Then you can plug x in to either original equation to get y:

$$\begin{aligned} 2x+y &= 3 \\ 2\cdot 2+y &= 3 \\ 4+y &= 3 \\ y &= -1 \end{aligned}$$

Then check by plugging x and y in to the original equations. For example: $2x+y=2\cdot 2+(-1)=3\checkmark$

6.2 Example

Solve:

$$\begin{aligned} 2x-4y &= 13 \\ 4x-5y &= 8 \end{aligned}$$

Solution: Multiply both sides of one equation so, when added to the other equation, one variable goes away:

$$\begin{aligned} -2(2x-4y) &= -2\cdot 13 \\ 4x-5y &= 8 \end{aligned}$$

The point of this is to get rid of the x term, which we see when we simplify...

$$\begin{array}{rcl} -4\,x+8\,y &=& -26 \\ 4\,x-5\,y &=& 8 \\ \text{———} && \text{— –} \\ 3\,y &=& -18 \\ y &=& -6 \end{array}$$

...then use substitution for the first variable in either of the first equations:

$$\begin{array}{rcl} 13 &=& 2\,x-4\,y \\ &=& 2\,x-4\cdot(-6) \\ &=& 2\,x+24 \\ -11 &=& 2\,x \\ -\frac{11}{2} &=& x \end{array}$$

So the solution is $(-\frac{11}{2},-6)$.

Check: See if that really is a solution by plugging in to the equations. For example:

$$\begin{array}{rcl} 4\,x-5\,y &=& 4(-\frac{11}{2})-5\,(-6) \\ &=& -22+30 \\ &=& 8\checkmark \end{array}$$

6.3 Example: the hybrid car

$$\begin{array}{rcl} 40\,x+2\,y &=& 200 \\ 3\,x+.1\,y &=& 13 \end{array}$$

Let's multiply both sides of the 2nd equation by -20 to get the y terms to disappear:

$$\begin{array}{rcll} 40\,x+2\,y &=& 200 & \\ -60\,x+(-2)y &=& -260 & \text{then, adding:} \\ \text{—————} && \text{——} & \\ -20\,x &=& -60 & \\ x &=& 3 & \end{array}$$

So we need 3 gallons of gas. Then we can substitute to find how much electric charge we need:

$$\begin{array}{rcl} 40\,x+2\,y &=& 200 \\ 40\cdot3+2\,y &=& 200 \\ 120+2\,y &=& 200 \\ 2\,y &=& 80 \\ y &=& 40 \end{array}$$

So we need 40 kw-h of electricity. We can check: $3\,x+.1\,y=3\cdot3+.1\cdot40=9+4=13\checkmark$ Also, $40\,x+2\,y=40\cdot3+2\cdot40=120+80=200\checkmark$.

6.4 Example

You can multiply *each* equation by a number and add. For example:

Solve:

$$\begin{array}{rcl} 7\,x-12\,y &=& -22 \\ -5\,x+8\,y &=& 14 \end{array}$$

We can make the y term coefficients opposites this way:

$$\begin{aligned} 2(7x-12y) &= 2(-22) \\ 3(-5x+8y) &= 3(14) \end{aligned}$$

Using the distributive law:

$$\begin{aligned} 14x-24y &= -44 \\ -15x+24y &= 42 \\ \hline -1x &= -2 \\ x &= 2 \end{aligned}$$

Then we can solve for y using one of the other equations. Of course we could have just multiplied the 2nd equation by $\frac{3}{2}$, but that would have involved fractions.

7 No solution

Sometimes a system has no solution:

$$\begin{aligned} x-2y &= 3 \\ 2x-4y &= 7 \end{aligned}$$

Try the linear combination method; multiply both sides of the 1st equation by -2:

$$\begin{aligned} -2x+4y &= 3 \\ 2x-4y &= 7 \\ \hline 0 &= 10 \end{aligned}$$

This obviously has no solution (no (x, y) pair can make it true).

If instead we had ended up with something like

$$3=3$$

we would know we had infinitely many solutions.