Section 2.4: Equations of Lines

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Algebra 2 Lv 1

1 Warmup

1. Welcome/expectations signatures on desk
2. Got calculators?
3. Write down homework: Read Chap 2.3 (p. 82-94). Problems (p. 95-98): 1-12, 13-57 odd, 59, 62, 66, 69, 70, 75, 79, 83, 89, 91
4. How are graphs of functions and graphs of equations different? How are they similar?

2 Quiz soon

Not sure exactly when, but be ready...

3 Slope-intercept again

$$y = m\,x + b$$

For example, if $m = 2$ and $b = -1$, our equation is

$$y = 2\,x + (-1)$$

Reminders:

- m is the slope
- b is the y-intercept
- If we know the slope and y-intercept we know the equation
- If we know the equation (in this form) we know the slope and y-intercept, and therefore the graph

4 Point-slope form

But what if we just know the slope and some other point, (x_0, y_0), on the graph, and we don't know the y-intercept? Then we can write the equation in the *point-slope* form:

$$y - y_0 = m\,(x - x_0)$$

How do we know that this equation give the proper graph? Since two points determine a line we just have to show that two solutions to the equation are on the graph.

For our first point, let's pick the original point (x_0, y_0). Is it a solution to the equation (does it make the equation true)? Let's plug it in:

$$\begin{aligned} y - y_0 &= m\,(x - x_0) \\ y_0 - y_0 &= m\,(x_0 - x_0) \\ 0 &= 0 \end{aligned}$$

What other point is on the graph? Well, if we add 1 to x we should add m to y (why?)

$$\begin{aligned} y - y_0 &= m\,(x - x_0) \\ (y_0 + m) - y_0 &= m\,((x_0 + 1) - x_0) \\ m &= m \end{aligned}$$

So that shows that this equation does indeed produce the line with slope m passing through the point (x_0, y_0).

(show graphical example here)

If we know the point-slope form we can convert to slope-intercept to find the y-intercept:

$$\begin{aligned} y - y_0 &= m\,(x - x_0) \\ y &= m\,(x - x_0) + y_0 \\ &= m\,x - m\,x_0 + y_0 \\ &= m\,x + (-m\,x_0 + y_0) \end{aligned}$$

So, starting with the slope-intercept form we can determine that the y-intercept is

$$-m\,x_0 + y_0$$

5 Equations of parallel and perpendicular lines

If you have a linear equation (in slope-intercept, point-slope or whatever) and want an equation of a parallel line through the point (x_1, y_1), just find the slope m and use the point-slope form:

$$y - y_1 = m\,(x - x_1)$$

How about a line perpendicular to the first one? Remember that if a line has a slope of m, perpendiculars have a slope of $-\frac{1}{m}$.

6 Just have two points

If you just have two points (x_0, y_0) and (x_1, y_1), you can find the slope

$$\frac{\text{rise}}{\text{run}} = \frac{y_1 - y_0}{x_1 - x_0}$$

Then you can use any of the above techniques.

7 The real world: CO_2 in the atmosphere

In 1960 there were about 320 ppm of CO_2 in the atmosphere. In 2000 there were about 370 ppm. *If it keeps growing linearly*, in what year will the concentration have doubled (to 640) from 1960 levels?

$$\begin{aligned} (x_0, y_0) &= (1960, 320) \\ (x_1, y_1) &= (2000, 370) \\ \text{slope} &= \frac{y_1 - y_0}{x_1 - x_0} \\ &= \frac{370 - 320}{2000 - 1960} \\ &= \frac{5}{4} \end{aligned}$$

Then an equation for the line is (using point-slope):

$$y - 320 = \frac{5}{4}(x - 1960)$$

Solving for $y = 640$ we get

$$\begin{aligned} 640 - 320 &= \frac{5}{4}(x - 1960) \\ 320 &= \frac{5}{4}x - 2450 \\ 320 + 2450 &= \frac{5}{4}x \\ 2770 &= \frac{5}{4}x \\ 2216 &= x \end{aligned}$$

8 Direct variation

Back to slope-intercept, there's a special case where the y-intercept is 0. In that case we can write the slope-intercept form as

$$y = m\,x$$

In that case we sometimes rename the slope constant k instead of m and we call it the *constant of variation*, and we say that y *varies directly* with x. We may write it in two equivalent forms:

$$\begin{aligned} y &= k\,x \\ \frac{y}{x} &= k \end{aligned}$$