Point/Slope Form Again

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Algebra 2 Lv 1

Problem: Find an equation of the line of slope 3 going through the point (1, 2). One way would be to force the information into slope-intercept form by solving for b:

$$\begin{aligned} 2 &= 3(1)+b \\ 2-3 &= b \\ -1 &= b \end{aligned}$$

From this we have the equation

$$y = 3x - 1$$

We can check that this is right by noting that the slope is right (since we are told it is 3) and it goes through the point (1, 2) because $2 = 3 \cdot 1 - 1$.

But there's an easier way! Because there is only one line of a given slope that goes through a given point, if we can find a linear equation with the right slope that goes through that point, we are done. And *going through* a point means that point is a solution to the equation. Now look at this equation:

$$y - 2 = 3(x - 1)$$

You can see immediately that this has slope 3. Does it go through the point (1, 2)? Try plugging it in:

$$\begin{aligned} 2-2 &= 3(1-1) \\ 0 &= 0 \end{aligned}$$

So we are done.

1 Point/slope general form.

That was one case; here is the general form. If we just know the slope and some point, (x_0, y_0), on the graph, and we don't know the y-intercept? Then we can write the equation in the *point-slope* form:

$$y - y_0 = m(x - x_0)$$

How do we know that this equation gives the proper graph? It is easy to see that it has the proper slope, m. It is also easy to see that (x_0, y_0) is a solution by just plugging those values in for x and y. So we are done!

(show graphical example here)

2 Examples

Example. Find the equation of a line of slope $\frac{2}{3}$ that includes the point $(1, -4)$.

Solution. All we do is plug those numbers into the form:

$$y - (-4) = \frac{2}{3}(x - 1)$$

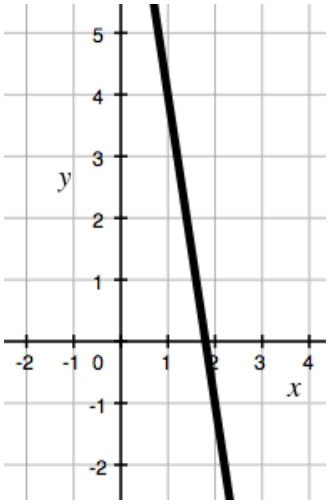
Done!

Example. Draw a graph of

$$y - (3) = 4(x - (-2))$$

Solution. It's in point-slope form so we know the graph includes the point $(-2, 3)$ and the slope is 4, so we should know what to do.

Example. Write an equation whose solution is this graph:



Solution. We can see from the graph that the point $(1, 4)$ is a solution and that the slope is -5. So we can mindlessly use the point-slope form:

$$y - 4 = -5(x - 1)$$

3 Given only two points

What if we are given two points but not the slope? Then just compute the slope from both points and use that slope and one of those points in the point-slope formula.

4 Converting to slope-intercept form

If we know the point-slope form we can convert to slope-intercept to find the y-intercept:

$$\begin{aligned} y - y_0 &= m(x - x_0) \\ y &= m(x - x_0) + y_0 \\ &= mx - mx_0 + y_0 \\ &= mx + (-mx_0 + y_0) \end{aligned}$$

So, starting with the point/slope form we can determine that the y-intercept is

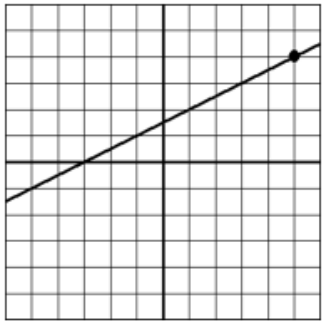
$$-m\,x_0 + y_0$$

1 Problems

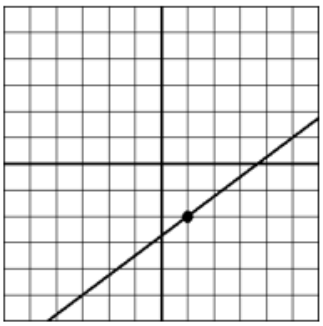
Write an equation in point/slope form, sketch a graph using point/slope form, and then rewrite in slope-intercept form:

1. A line of slope -2 going through the point $(1, -3)$
2. A line of slope $\frac{2}{3}$ going through the point $(1, \frac{1}{2})$
3. A line perpendicular to line 2 going through the same point
4. A line of slope 1 going through the point $(1, 1)$
5. A line perpendicular to line 4 going through the same point
6. A line of slope 0 going through the point $(1, -1)$
7. Can you use the point/slope formula to get a line perpendicular to line 6 going through the same point? Why or why not?
8. Regardless of problem 7, what equation gives the line requested in line 7?
9. A line of slope 3 through the point (0, 2). Why are the slope/intercept and point/slope forms so similar?
10. A line of slope m going through the point (x_0, y_0) (this is the general case; do not graph)
11. Write an equation in point/slope form for the following graphs:

a)



b)



12. Consider the (non-linear) function $f(x) = x^2$. In calculus you will learn that the slope of the line tangent to that particular curve for $x = x_0$ is $2\,x$. The tangent line for $x = x_0$ also touches the curve so it must go through the point $(x_0, f(x_0))$. So for $x = 1$, the slope of the tangent line is $2 \cdot 1$ and the tangent line must go through the point $(1, 1^2)$.

    a) Using graph paper, graph f for x values between -3 and 3. (Just plot $f(-3)$, $f(-2)$, ..., $f(2)$, $f(3)$ and connect with a smooth curve.)

    b) Using the point/slope form, compute an equation for that tangent line at $x = 1$ (its slope is 2 and it must go through the point $(1, 1)$).

    c) Graph that tangent line on the same graph.

    d) Using the point/slope form again, compute an equation for that tangent line at $x = 2$.

    e) Graph that tangent line.

    f) Compute the equation (still point/slope form) and graph the tangent line at $x = 0$.

13. In 1960 there were about 320 ppm of CO_2 in the atmosphere. In 2000 there were about 370 ppm. *If it keeps growing linearly*, in what year will the concentration have doubled (to 640) from 1960 levels?