

Algebra 2 Final Exam Review 2: Solving Equations and Inequalities

Notes: There are many types of equations we've solved this year, but many/most of them are basically the same. Here are a few types of equations that you need to handle algebraically:

1. Most equations with only one x in them. Solve these by "isolate and undo". The equations below look different but fall into this camp, since they have only one x in them.

$$3x - 1 = 12 \qquad 5(x - 2)^2 + 7 = 18 \qquad 3|x - 2| + 5 = 31 \qquad -2\log(x - 3) - 1 = -3$$

Solve these by isolating the expression with the x in it (by adding, subtracting, multiplying, or dividing) and then undoing the operation by doing the opposite. To undo a square, take the square root of both sides; to undo a cube root, cube both sides; to undo an absolute value, take the positive or negative of the other side. To undo any power, take both sides to the reciprocal power. To undo a logarithm, write as an exponential equation.

2. Polynomial equations with more than one x in them. These can usually be solved by factoring or the quadratic formula. Be sure to set one side to zero before doing any factoring.
3. Other types of equations can usually be manipulated into one of these types by doing things to "get rid" of the other mathematical operations. These include equations such as:

$$\sqrt{x - 2} + 1 = x - 5 \qquad 2^x \cdot 4^{x^2} = 16 \qquad \log_2 x + \log_2(x - 2) = 3 \qquad \frac{3}{x} - \frac{x + 1}{x - 5} = 11$$

-In the first one, isolate the radical and square both sides.

-In the second one, write each side as 2 to some power and set the exponents equal to each other.

-In the third one, condense the logs and rewrite as an exponential equation.

-In the fourth one, multiply through by the LCD of $x(x - 5)$

In each case you will have a quadratic or linear equation that can be solved by factoring or the quad formula.

4. Equations where x is in the exponent (exponential equations). If you cannot find a common base and solve as another type of equation, you need to take the log of both sides.

$$3^x = 31$$

Common themes

-remove fractions first (even if they have x 's)

-look for multiple solutions (undoing absolute values, square roots, 4th roots)

-check for extraneous solutions

Domain issues: root of negative, divide by zero, log of zero or negative

When permissible you can usually find the real solutions with calc-intersect (when graphing both sides) or calc-zero (when one side of the equation is zero).

Solving equations also comes up in these types of questions:

-**finding zeros or x-intercepts** (remember, x-intercepts are real zeros)

-**finding intersections:** substitution is often easiest; if both are solved for y set them equal to each other (remember to find the y -coordinate!)

-**solving inequalities:** for most of them, solve the related equation and then check which parts of the number line it is true on (between and around points).

Problems

1. Find all real solutions to the following equations using “isolate and undo”. Be sure to check for extraneous solutions and watch for multiple solutions (when undoing even powers and absolute values). You should only need your calculator for ones marked with *.

a. $2x - 7 = 11$

b. $3(x + 2)^2 - 5 = 67$

c. $|x - 3| + 1 = 5$

d. $\log_3(x - 3) - 3 = -1$

e. $\frac{2}{3}x - 1 = x - \frac{3}{5}$

f. $2|x - 2| + 5 = 3$

g. $x^{2/3} - 7 = 2$

h. $x^2 - 7 = 13$

i. $3(x + 1)^{1/3} - 2 = 3$

j*. $10(1 + x)^{12} = 45$

k. $2\sqrt{2x - 1} = 6$

l. $2(x + 4)^{-1} = 6$

m. $\frac{3}{x - 1} + 2 = 1$

n. $2 - 3\sqrt{x} = 7$

o. $3(x + 6)^3 + 8 = 5$

p. $2\log_4(x - 1) + 1 = 5$

q.* $20x^{11} + 5 = 143$

2. Find all real and imaginary/complex solutions to the following equations by factoring or using the quadratic formula.

a. $2x^2 - 3x = x + 30$

b. $2(x + 3)^2 - 10(x + 2) = x^2 + 4x + 6$

c. $(x - 2)(x + 3) = 6$

d. $x^2 + x + 7 = 0$

e. $x^3 + 6x^2 = 3x$

f. $x^4 - 7x^2 = 8$

g. $x^3 + 6x^2 + 3x + 18 = 0$

h. $\frac{1}{3}x^4 + \frac{2}{3}x^3 = 8x^2$

i. $-2(x + 3)^2 - 5 = 11$

j. $39(2x - 1)(x^2 - 7) = 0$

3. Find all real solutions to the following equations. You will first have to perform operations to make these look like polynomial equations that you can solve by isolating, factoring, or using the quadratic formula.

a. $\frac{x + 3}{x} = \frac{x + 1}{x - 1}$

b. $\log(x - 3) = \log(2x - 5)$

c. $3^{2x-1} = \frac{1}{9}$

d. $3\sqrt{x} = x + 2$

e. $\log_3 x - \log_3 4 = -1$

f. $\sqrt[3]{6x^2} = 2x$

g. $\frac{3}{x} + 1 = \frac{-9}{x - 12}$

h. $\frac{1}{2}\log_3(x - 2) - \log_3 4 = 1$

i. $4^x = 2\sqrt{2}$

j. $\frac{x + 1}{x - 3} + 1 = \frac{8}{x - 3}$

k. $\log_4 x + \log_4(x + 2) = \log_4 3$

l. $4^{3x} = 16^{x+2}$

4. Solve the following equations. You will need to isolate the term with the x and take the log of both sides.

a. $7^x = 31$

b. $2 \cdot 3^x - 7 = 81$

c. $10^x = 0.18$

5. Identify the shape of the graph of each function below and then algebraically find the exact coordinates of all points of intersection. Where both graphs are lines, linear combination or substitution will work; otherwise I recommend substitution.

a. $y = 2x - 1$
 $2x + y = 7$

b. $x + 3y = 13$
 $2x - y = 5$

c. $y = 2x^2 - 5$
 $y = 8x - 5$

d. $y = \frac{1}{2}x^3 - 2$
 $y = 2$

e. $y = \sqrt{x+3}$
 $y = x + 1$

f. $y = 2x + 7$
 $4x - 2y = 11$

6. Solve the following inequalities. I recommend that you solve the related equation first and then check which segments of the number line the inequalities holds for.

a. $|x - 4| < 3$

b. $(x - 5)(2x + 7) \geq 0$

c. $-x^2 + 9x < 0$

d. $7 - 2|x + 1| \leq -3$

7. Use calc-intersect or calc-zero to solve the following equations and inequalities:

a. $|2x - 1| = x + 3$

b. $x^3 - 2x^2 + 5 = 2x + 7$

c. $3\sqrt{x^2 - 5} = 12 - \frac{1}{2}x^2$

d. $2\sqrt{x+6} - 3 \geq (x-2)^2 - 5$

Answers

1a. 9 b. $-2 \pm 2\sqrt{6}$ c. 7, -1 d. 12 e. -6/5 f. none g. 27, -27 h. $\pm 2\sqrt{5}$ i. 98/27

j. 0.134, -2.134 k. 5 l. -11/3 m. -2 n. none o. -7 p. 17 q. 1.192

2a. 5, -3 b. 4, -2 c. 3, -4 d. $\frac{(-1 \pm 3i\sqrt{3})}{2}$ e. $0, -3 \pm 2\sqrt{3}$ f. $\pm 2\sqrt{2}, \pm i$

g. -6, $\pm i\sqrt{3}$ h. 0, 4, -6 i. $-3 \pm 2i\sqrt{2}$ j. $\frac{1}{2}, \pm\sqrt{7}$

3a. 3 b. none ($x=2$ is extraneous) c. $-\frac{1}{2}$ d. 4, 1 e. 4/3 f. $0, \frac{3}{4}$ g. 6, -6 h. 146

i. $\frac{3}{4}$ j. 5 k. 1 (-3 is extraneous) l. 4

4a. 1.765 b. 3.445 c. -0.745

5a. both lines (2,3) b. lines (4,3) c. parabola and line (0,-5) and (4,27)

d. cubic and horiz line (2,2) e. sq root and line; (1,2) f. lines: none (parallel)

6a. $1 < x < 7$ b. $x \geq 5$ or $x \leq -3.5$ c. $x < 0$ or $x > 9$ d. $x \leq 4$ or $x \geq 6$

7a. -2/3, 4 b. 2.920 c. 3.202, -3.202 d. $-0.580 \leq x \leq 4.935$