Algebra 2 Final Review Part 1: Functions

Summary Notes

A **function** defines a relationship or mapping between an input variable and an output variable. For a relation to be a function, no input may have more than one output. Graphically, this is represented by the **vertical line test**. A function may be represented by an input-output table, an algebraic rule, or a graph. The set of inputs of a function is called the **domain**, the set of outputs is called the **range**. The **zeros** of a function are all inputs that cause the output to be zero. Graphically, this is an x-intercept (although a function may have imaginary/complex zeros that do not show up as x-intercepts).

Domains can be found graphically or algebraically. Algebraically, with the functions we have studied this year, the only **domain exceptions** are inputs caused one to: divide by zero, take the square-root of a negative number, or take the logarithm of a negative number or zero.

Functions can be put together in various ways to build more functions. We can add, subtract, multiply, or divide function outputs. Notationally, these are represented by $(f+g)(x)$, $(f-g)(x)$, $(f\cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ respectively.

Inverse Functions: An inverse function switches the input and output. A function has an inverse function if it passes the horizontal-line test. You can find the inverse function (denoted by $f^{-1}(x)$) by replacing $f(x)$ with y, switching the x's and y's and then solving for y.

Questions

1. Use the graph of the function $f(x)$ to answer the following questions:

a. What is the domain of $f(x)$?

b. What is the range of $f(x)$?

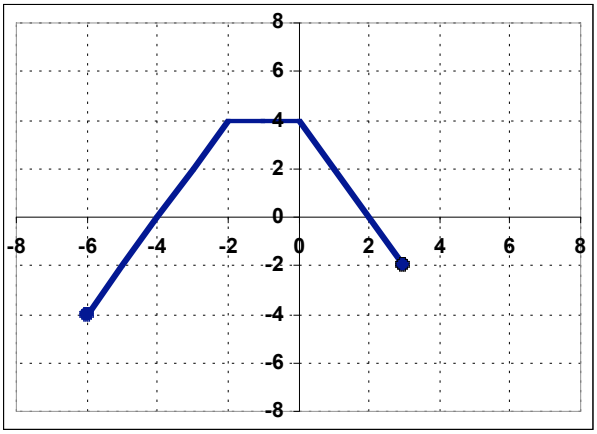
c. What are the zeros of $f(x)$?

d. What is $f(-2)$?

e. What are the solutions to $f(x) = 2$?

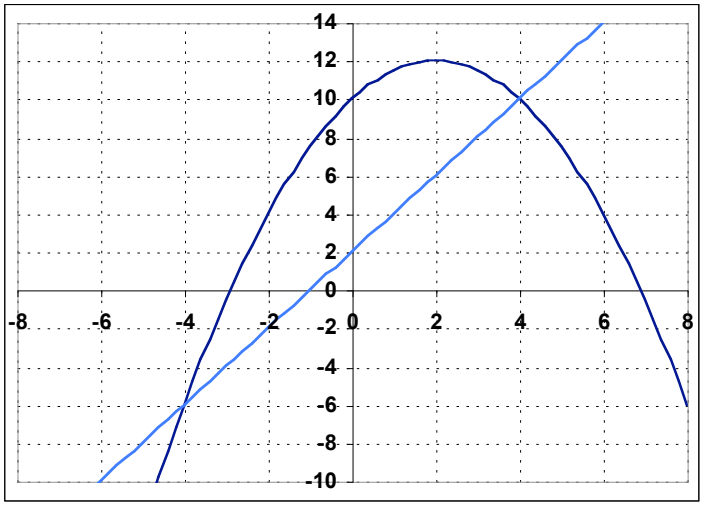
f. Write an inequality showing the solution to $f(x) > 2$.

g. Write an inequality showing the solution to $f(x) < 0$.



2. The parabola $f(x)$ and line $g(x)$ are graphed below. **The domains of the functions are all real numbers, only a portion of their graphs are shown.** Use their graphs to answer the following:

a. What is the range of $f(x)$?
b. What is the range of $g(x)$?
c. What are the zeros of $f(x)$ and $g(x)$?
d. Estimate the solutions to $f(x) = -2$.
e. What is $g(-6)$?
f. What is $f(6)$?
g. What are the solutions to $f(x) = g(x)$?
h. What is the solution to $g(x) \le 0$? (inequality)
i. What is the solution to $f(x) \le g(x)$? (ineq)
j. What is $g(f(-3))$?



3. Find the domain of the following functions algebraically:

a. $f(x) = \sqrt{x+5} - 1$

b. $f(x) = \dfrac{5}{2x-8}$

c. $f(x) = \dfrac{2x-1}{x^2+6x}$

d. $f(x) = -2\sqrt{2x-7} + 3$

e. $f(x) = \log_5(x-2) + 3$

4. Find the zeros of the following functions algebraically. One may require the quadratic formula.

a. $f(x) = \dfrac{2}{3}x - 7$

b. $f(x) = 3(x-2)(x+7)$

c. $f(x) = \sqrt{x-3} - 4$

d. $f(x) = \dfrac{3}{x} - 6$

e. $f(x) = x^2 - 6x + 3$

f. $f(x) = 3(x-1)^2 - 12$

5. Find inverses of each of the following functions

a. $f(x) = 3x - 2$

b. $f(x) = (x-4)^3 + 2$

c. $g(x) = 2 \cdot \sqrt[5]{x+3} - 7$

6. Given that $f(x) = 2x - 1$ and $g(x) = x^2 - 5$, find the following:

a. $(f-g)(x)$

b. $(f+g)(-2)$

c. $(f \cdot g)(0)$

d. $g(i)$

e. $g(1+i)$

7. Given that $f(x) = 2x^2$, $g(x) = 2x^{-1}$, and $h(x) = \sqrt{6x}$, find the following. Simplify all answers (no negative exponents and all radicals should be in simplest radical form).

a. $h(8)$ b. $g(1/3)$ c. $h(3) + h(12)$ d. $h(3) \cdot h(12)$

8. The function $f(x)$ below is a piece-wise function, meaning that different rules apply for different parts of the domain.

$$f(x) = \begin{cases} 2x + 3 & x < -3 \\ -x^2 + 5 & -3 \le x < 5 \\ 8 & x \ge 5 \end{cases}$$

Find the following:

a. $f(-4)$ b. $f(-3)$ c. $f(7)$

d. $f(5)$ e. the y-intercept of $f(x)$

Answers

1a. $-6 \le x \le 3$ b. $-4 \le y \le 4$ c. -4, 2 d. 4 e. -3, 1 f. $-3 < x < 1$
g. $-6 \le x < -4$ *or* $2 < x \le 3$

2a. $y \le 12$ b. all reals c. $f(x)$: -3, 7 $g(x)$: -1 d. about -3.3, 7.3 e. -10 f. 4
g. x=-4, 4 h. $x \le -1$ i. $x \le -4$ *or* $x \ge 4$ j. this is $g(0)$ which is 2

3a. $x \ge -5$ b. all reals $x \ne 4$ c. all reals $x \ne 0, -6$ d. $x \ge 3.5$ e. $x > 2$

4a. 10.5 b. 2, -7 c. 19 d. ½ e. $3 \pm \sqrt{6}$ f. -1, 3

5a. $f^{-1}(x) = \dfrac{x+2}{3}$ b. $f^{-1}(x) = \sqrt[3]{x-2} + 4$ c. $g^{-1}(x) = \left(\dfrac{x+7}{2}\right)^5 - 3$

6a. $-x^2 + 2x + 4$ b. -6 c. 5 d. -6 e. $-5 + 2i$

7a. $4\sqrt{3}$ b. 6 c. $9\sqrt{2}$ d. 36

8a. -5 b. -4 c. 8 d. 8 e. (0,5)