12.5 Probability of Independent and Dependent Events

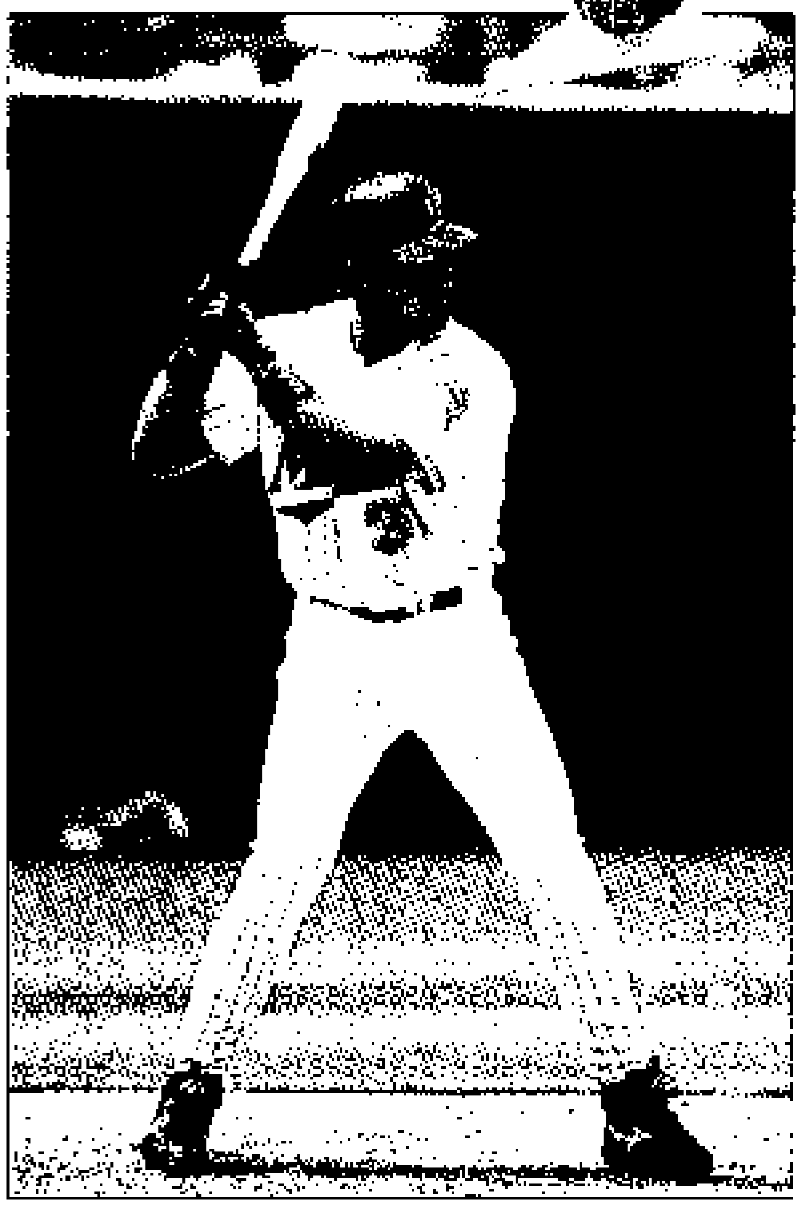
What you should learn

GOAL 1 Find the probability of independent events.

GOAL 2 Find the probability of dependent events, as applied in **Ex. 33**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the probability that the Florida Marlins win three games in a row in **Example 2**.



GOAL 1 PROBABILITIES OF INDEPENDENT EVENTS

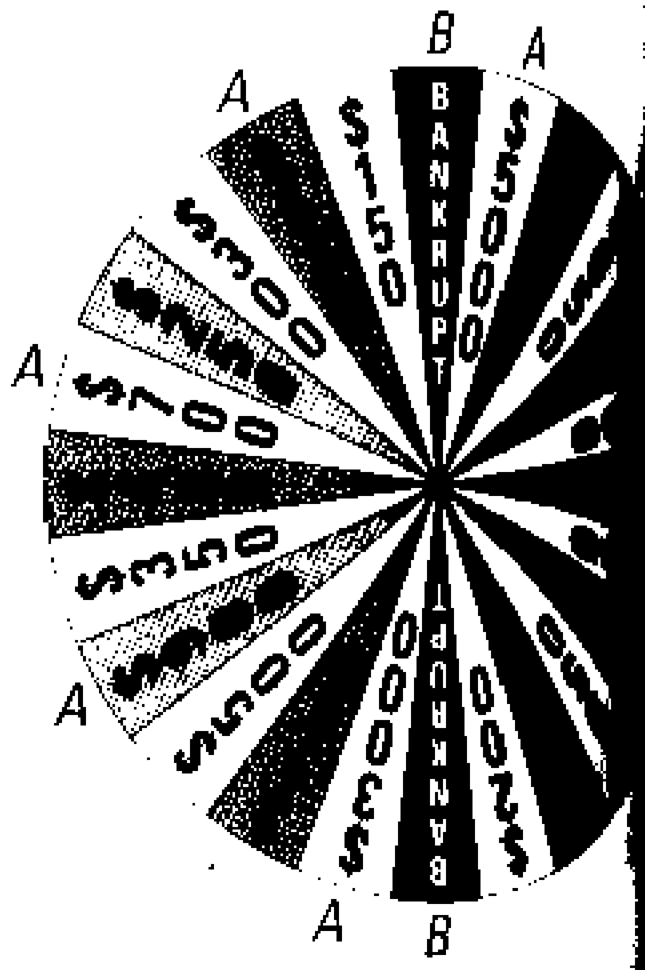
Two events are **independent** if the occurrence of one has no effect on the occurrence of the other. For instance, if a coin is tossed twice, the outcome of the first toss (heads or tails) has no effect on the outcome of the second toss.

PROBABILITY OF INDEPENDENT EVENTS

If A and B are independent events, then the probability that both A and B o is $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 1 *Probability of Two Independent Events*

You are playing a game that involves spinning the money wheel shown. During your turn you get to spin the wheel twice. What is the probability that you get more than \$500 on your first spin and then go bankrupt on your second spin?



SOLUTION Let event A be getting more than \$500 on the first spin, and let event B be going bankrupt on the second spin. The two events are independent. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{8}{24} \cdot \frac{2}{24} = \frac{1}{36} \approx 0.028$$

The formula given above for the probability of two independent events can be extended to the probability of three or more independent events.

EXAMPLE 2 *Probability of Three Independent Events*

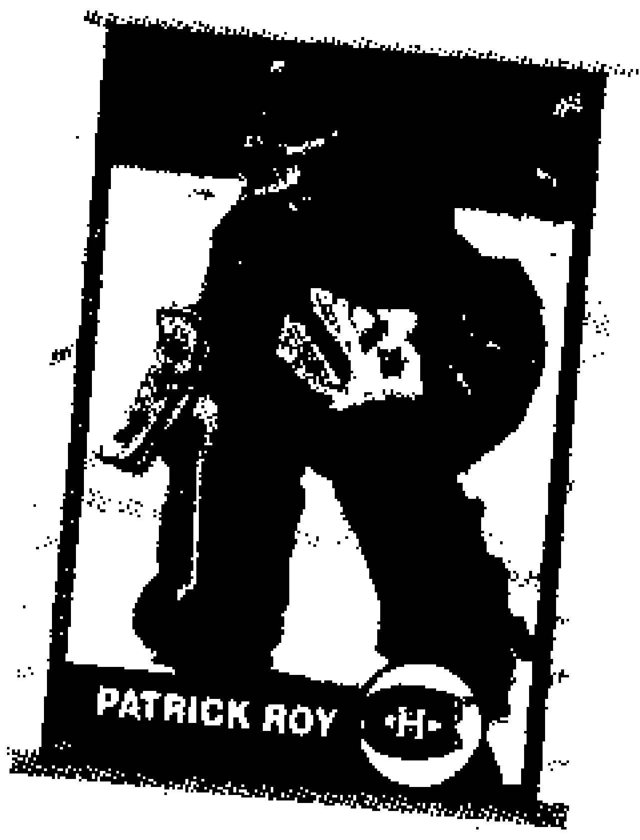
BASEBALL During the 1997 baseball season, the Florida Marlins won 5 out of 7 home games and 3 out of 7 away games against the San Francisco Giants. During t 1997 National League Division Series with the Giants, the Marlins played the first two games at home and the third game away. The Marlins won all three games. Estimate the probability of this happening. ▶ Source: The Florida Marlins

SOLUTION Let events A, B, and C be winning the first, second, and third games. The three events are independent and have experimental probabilities based on the regular season games. So, the probability of winning the first three games is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B) \cdot P(C) = \frac{5}{7} \cdot \frac{5}{7} \cdot \frac{3}{7} = \frac{75}{343} \approx 0.219$$

Trading Cards

EXAMPLE 3 *Using a Complement to Find a Probability*



You collect hockey trading cards. For one team there are 25 different cards in the set, and you have all of them except for the starting goalie card. To try and get this card, you buy 8 packs of 5 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get at least one starting goalie card.

SOLUTION

In one pack the probability of *not* getting the starting goalie card is:

$$P(\text{no starting goalie}) = \frac{{}_{24}C_5}{{}_{25}C_5}$$

Buying packs of cards are independent events, so the probability of getting at least one starting goalie card in the 8 packs is:

$$\begin{aligned} P(\text{at least one starting goalie}) &= 1 - P(\text{no starting goalie in any pack}) \\ &= 1 - \left(\frac{{}_{24}C_5}{{}_{25}C_5}\right)^8 \\ &\approx 0.832 \end{aligned}$$

Manufacturing

EXAMPLE 4 *Solving a Probability Equation*

A computer chip manufacturer has found that only 1 out of 1000 of its chips is defective. You are ordering a shipment of chips for the computer store where you work. How many chips can you order before the probability that at least one chip is defective reaches 50%?

SOLUTION

NT HELP

Look Back For help with solving exponential equations, see p. 501.

Let n be the number of chips you order. From the given information you know that $P(\text{chip is } not \text{ defective}) = \frac{999}{1000} = 0.999$. Use this probability and the fact that each chip ordered represents an independent event to find the value of n.

$P(\text{at least one chip is defective}) = 0.5$	**Write given assumption.**
$1 - P(\text{no chips are defective}) = 0.5$	**Use complement.**
$1 - (0.999)^n = 0.5$	**Substitute known probability.**
$-(0.999)^n = -0.5$	**Subtract 1 from each side.**
$(0.999)^n = 0.5$	**Divide each side by –1.**
$n = \frac{\log 0.5}{\log 0.999}$	**Solve for n.**
$n \approx 693$	**Use a calculator.**

▶ If you order 693 chips, you have a 50% chance of getting a defective chip. Therefore, you can order 692 chips before the probability that at least one chip is defective reaches 50%.

GOAL 2 PROBABILITIES OF DEPENDENT EVENTS

Two events A and B are **dependent events** if the occurrence of one affects the occurrence of the other. The probability that B will occur given that A has occurred is called the **conditional probability** of B given A and is written $P(B \mid A)$.

PROBABILITY OF DEPENDENT EVENTS

If A and B are dependent events, then the probability that both A and B occur is $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$.

Endangered Species

EXAMPLE 5 *Finding Conditional Probabilities*

The table shows the number of endangered and threatened animal species in the United States as of November 30, 1998. Find **(a)** the probability that a listed animal is a reptile and **(b)** the probability that an endangered animal is a reptile.

▶ Source: United States Fish and Wildlife Service

	Mammals	Birds	Reptiles	Amphibians	Other
Endangered	59	75	14	9	198
Threatened	8	15	21	7	69

SOLUTION

a. $P(\text{reptile}) = \dfrac{\text{number of reptiles}}{\text{total number of animals}} = \dfrac{35}{475} \approx 0.0737$

b. $P(\text{reptile} \mid \text{endangered}) = \dfrac{\text{number of endangered reptiles}}{\text{total number of endangered animals}}$

$$= \frac{14}{355} \approx 0.0394$$

EXAMPLE 6 *Comparing Dependent and Independent Events*

STUDENT HELP

Look Back
For help with a standard 52-card deck, see p. 708.

You randomly select two cards from a standard 52-card deck. What is the probability that the first card is not a face card (a king, queen, or jack) and the second card is a face card if **(a)** you replace the first card before selecting the second, and **(b)** you do *not* replace the first card?

SOLUTION

a. If you replace the first card before selecting the second card, then A and B are independent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B) = \frac{40}{52} \cdot \frac{12}{52} = \frac{30}{169} \approx 0.178$$

b. If you do *not* replace the first card before selecting the second card, then A and B are dependent events. So, the probability is:

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A) = \frac{40}{52} \cdot \frac{12}{51} = \frac{40}{221} \approx 0.181$$

The formula for finding probabilities of dependent events can be extended to three or more events, as shown in Example 7.

REAL LIFE
Dining Out

EXAMPLE 7 *Probability of Three Dependent Events*

You and two friends go to a restaurant and order a sandwich. The menu has 10 types of sandwiches and each of you is equally likely to order any type. What is the probability that each of you orders a different type?

SOLUTION

Let event A be that you order a sandwich, event B be that one friend orders a different type, and event C be that your other friend orders a third type. These events are dependent. So, the probability that each of you orders a different type is:

$$P(A \text{ and } B \text{ and } C) = P(A) \cdot P(B \mid A) \cdot P(C \mid A \text{ and } B)$$

$$= \frac{10}{10} \cdot \frac{9}{10} \cdot \frac{8}{10}$$

$$= \frac{18}{25} = 0.72$$

STUDENT HELP

Study Tip
You can also use the fundamental counting principle to find the probability in Example 7.

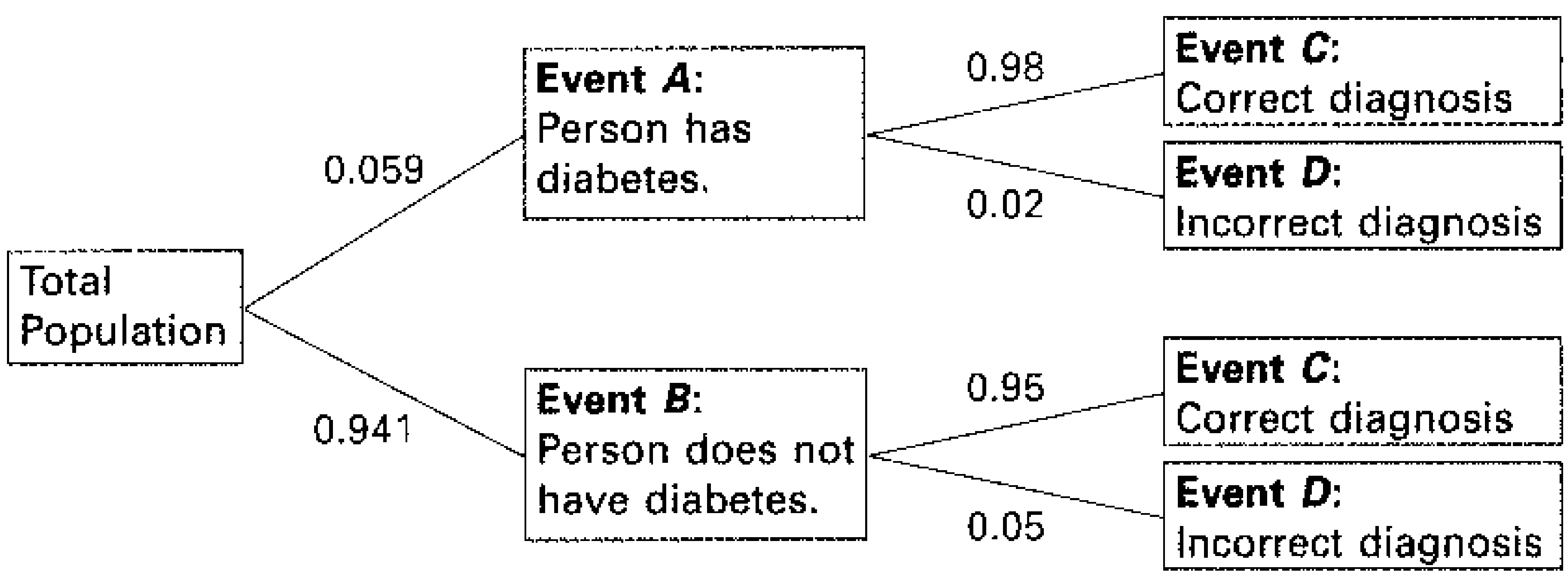
$$P(\text{all different}) = \frac{\text{no. of different orders}}{\text{no. of possible orders}} = \frac{10 \cdot 9 \cdot 8}{10 \cdot 10 \cdot 10} = 0.72$$

EXAMPLE 8 *Using a Tree Diagram to Find Conditional Probabilities*

HEALTH The American Diabetes Association estimates that 5.9% of Americans have diabetes. Suppose that a medical lab has developed a simple diagnostic test for diabetes that is 98% accurate for people who have the disease and 95% accurate for people who do not have it. If the medical lab gives the test to a randomly selected person, what is the probability that the diagnosis is correct?

SOLUTION

A probability tree diagram, where the probabilities are given along the branches, can help you see the different ways to obtain a correct diagnosis. Notice that the probabilities for all branches from the same point must sum to 1.



So, the probability that the diagnosis is correct is:

$P(C) = P(A \text{ and } C) + P(B \text{ and } C)$	**Follow branches leading to *C*.**
$= P(A) \cdot P(C \mid A) + P(B) \cdot P(C \mid B)$	**Use formula for dependent events.**
$= (0.059)(0.98) + (0.941)(0.95)$	**Substitute.**
≈ 0.952	**Use a calculator.**

GUIDED PRACTICE

Vocabulary Check ✓

1. Explain the difference between dependent events and independent events, and give an example of each.

1. Events are independent if the occurrence of one has no effect on the occurrence of the other. Events are dependent if the occurrence of one affects the occurrence of the other. *Sample answer:* Rolls of a die are independent events because the number you get on the first roll does not affect the number on the second roll. The sum of the numbers on two rolls of a die is dependent. For example, if the first roll is not a six, there is a 0 probability that the sum will be twelve.

Concept Check ✓

2. If event A is drawing a queen from a deck of cards and event B is drawing a king from the remaining cards, are events A and B dependent or independent? **dependent**

3. If event A is rolling a two on a six-sided die and event B is rolling a four on a different six-sided die, are events A and B dependent or independent? **independent**

Skill Check ✓

Events A and B are independent. Find the indicated probability.

4. $P(A) = 0.3$ **0.27**
$P(B) = 0.9$
$P(A \text{ and } B) = \underline{?}$

5. $P(A) = \underline{?}$ **0.2**
$P(B) = 0.3$
$P(A \text{ and } B) = 0.06$

6. $P(A) = 0.75$ **0.2**
$P(B) = \underline{?}$
$P(A \text{ and } B) = 0.15$

Events A and B are dependent. Find the indicated probability.

7. $P(A) = 0.1$ **0.08**
$P(B \mid A) = 0.8$
$P(A \text{ and } B) = \underline{?}$

8. $P(A) = \underline{?}$ **0.5**
$P(B \mid A) = 0.5$
$P(A \text{ and } B) = 0.25$

9. $P(A) = 0.9$ **0.6**
$P(B \mid A) = \underline{?}$
$P(A \text{ and } B) = 0.54$

READING LIST In Exercises 10 and 11, use the following information.
Three friends are taking an English class that has a summer reading list. Each student is required to read one book from the list, which contains 3 biographies, 10 classics, and 5 historical novels.

10. Find the probability that the first friend chooses a biography, the second friend chooses a classic, and the third friend chooses a historical novel. **0.026**

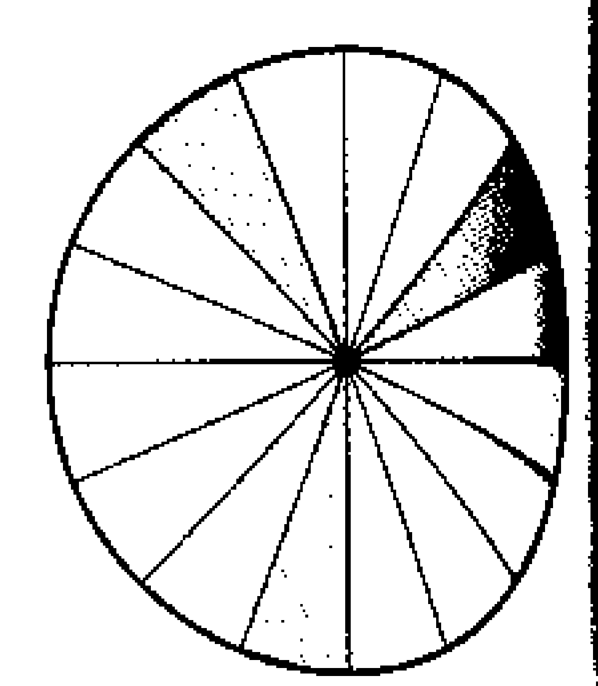
11. Find the probability that the three friends each choose a different classic. **0.123**

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 957.

SPINNING A WHEEL You are playing a game that involves spinning the wheel shown. Find the probability of spinning the given colors.



12. red, then blue **0.047**
13. red, then green **0.047**
14. yellow, then red **0.059**
15. green, then yellow **0.078**
16. blue, then yellow, then green **0.020**
17. green, then red, then blue **0.012**

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 12–15
Example 2: Exs. 16, 17, 24, 25
Example 3: Exs. 26, 27
Example 4: Exs. 28, 29
Example 5: Exs. 30–32
Example 6: Exs. 18–21
Example 7: Exs. 22, 23, 33, 34
Example 8: Exs. 35, 36

DRAWING CARDS Find the probability of drawing the given cards from a standard 52-card deck (a) with replacement and (b) without replacement.

18. a heart, then a diamond
a. 0.0625 b. 0.0637

19. a jack, then a king **a. 0.0059 b. 0.00**

20. a 2, then a face card (K, Q, or J)
a. 0.0178 b. 0.0181

21. a face card (K, Q, or J), then a 2
a. 0.0178 b. 0.0181

22. an ace, then a 2, then a 3
a. 0.00046 b. 0.00048

23. a heart, then a diamond, then another heart
a. 0.0156 b. 0.0153

24. **GAMES** You are playing a game that involves drawing three numbers from a hat. There are 25 pieces of paper numbered 1 to 25 in the hat. Each number is replaced after it is drawn. What is the probability that each number is greater than 20 or less than 4? **0.0328**

25. **LAWN CARE** The owner of a one-man lawn mowing business owns three old and unreliable riding mowers. As long as one of the three is working he can stay productive. From past experience, one of the mowers is unusable 12 percent of the time, one 6 percent of the time, and one 20 percent of the time. Find the probability that all three mowers are unusable on a given day. **0.00144**

26. **TRADING CARDS** You collect movie trading cards, which have different scenes from a movie. For one movie there are 90 different cards in the set, and you have all of them except the final scene. To try and get this card, you buy 10 packs of 8 cards each. All cards in a pack are different and each of the cards is equally likely to be in a given pack. Find the probability that you will get the final scene. **0.606**

27. **FREE THROWS** Chris Mullin of the Indiana Pacers led the National Basketball Association in free-throw percentage during the 1997–1998 season. He made 93.9% of his free-throw attempts. If he attempted 10 free throws in a game, what is the probability that he missed at least one? ▶ Source: NBA **0.467**

28. **MANUFACTURING** Look back at Example 4. Suppose the computer chip manufacturer has improved quality control so that only 1 out of 10,000 of its chips is defective. Now how many chips can you order before the probability that at least one chip is defective reaches 50%? **6931**

29. **LOTTERY** To win a state lottery, a player must correctly match six different numbers from 1 to 42. If a computer randomly assigns six numbers per ticket, how many tickets would a person have to buy to have a 1% chance of winning? **at least 52,722 tickets**

IDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with problem solving in Ex. 29.

FOCUS ON CAREERS

TEACHER In addition to ...ing, teachers plan daily ...ns and activities, ...n and correct home-..., and prepare and grade .... They also monitor ...ooms, study halls, and ...rias, meet with ...ts, and supervise ...curricular activities.

CAREER LINK ...mcdougallittell.com

STATISTICS CONNECTION **In Exercises 30 and 31, use the following information.** The table, based on a Gallup Poll, shows the number of voters (in 1000's) by party affiliation who were expected to vote for Bill Clinton and Bob Dole in the 1996 Presidential election. ▶ Source: The Gallup Organization

	Democrat	Republican	Independent
Clinton	31,378	3,340	12,685
Dole	2,092	28,386	8,721

30. Find the probability that a randomly selected person voted for Clinton. **0.547**

31. Find the probability that a randomly selected Democrat voted for Clinton. **0.937**

32. **TEACHERS** In the United States during the 1993–1994 school year, 39.6% of all male teachers and 26.1% of all female teachers had twenty years or more of full-time teaching experience. That year 694,000 males and 1,867,000 females were teachers. What is the probability that a randomly chosen teacher in the United States that year was a female with twenty years or more of full-time teaching experience? **0.19**

DATA UPDATE of *Statistical Abstract of the United States* data at www.mcdougallittell.com

33. **COSTUMES** You and four of your friends go to the same store at different times to buy costumes for a costume party. There are 20 different costumes at the store, and the store has at least five duplicates of each costume. Find the probability that all five of you choose different costumes. **0.581**

34. **AIRPLANE MEALS** On a long flight an airline usually serves a meal. If there are 2 choices for the meal, what is the probability that all 6 people in the first row choose the same meal assuming choices are made independently? **0.0156**