Review for Test: Logarithmic and Exponential Functions (Chapter 8 in book)

1. Sketch a graph of the following functions. Indicate the range, domain, and asymptote of each.

a. $f(x) = 3 \cdot 2^x - 1$ *b.* $f(x) = -2 \cdot \left(\frac{1}{2}\right)^x + 3$ *c.* $f(x) = -3 \cdot 4^{x-1} - 2$

2. Find the zero and y-intercept of each function below:

a. $f(x) = -\frac{3}{2} \cdot \left(\frac{1}{2}\right)^x + 6$ *b.* $f(x) = 2\log_2(x+4) - 6$

3. Solve the following exponential equations (without a calculator):

a. $4 \cdot 3^{x+1} = 108$ b. $2^{x+4} = 16^2$ c. $\left(\frac{1}{4}\right)^x = 8$

d. $3^{2x} = \left(\frac{1}{9}\right)^{x-3}$ e. $3 \cdot 4^{3x} - 6 = 42$ f. $2^2 \cdot 2^x = 8^{3-2x}$

g. $3^x \cdot 9^{x-2} = \frac{1}{27}$ h. $\frac{2^{x^2}}{2^{3x}} - 14 = 2$ i.. $\frac{4^{x^3}}{4^{3x}} = \left(\frac{4^{x^2}}{4^3}\right)^3$

4. Evaluate the following logarithms without calculator

a. $\log_5 125$ b. $\log_3 \frac{1}{9}$ c. $\log 5 + \log 20$ d. $\log_{1/2} 4$

e. $\log_4 \frac{1}{2}$ f. $2\log_6 2 + \log_6 9$ g. $\log_3 27\sqrt{3}$ h. $2\log_8 2 - \log_8 3 + \log_8 12$

4.5. Estimate the following:

a. log 106 b. $\log_2 15$ c. $\log_7 50$ d. log 99,999,994

5. Condense each of the following expressions into a single log:

a. $\log 5 + 2\log 3$

b. $\log 4 + 2\log 3 - 3\log 2$

c. $\log_4 18 - \log_4 6$

d. $2\log 5 - 5\log 2$

e. $2\log_7 2x + \log_7 y - 2\log_7 3$

f. $\log_3 2 - \frac{1}{2}\log_3 y^2$

g. $2(\log 6 - \log 2) + 3(\log 8 - \log 4)$

h. $\frac{1}{3}\log_5 27 + \frac{2}{3}\log_5 8 - \frac{3}{2}\log_5 9$

6. Expand the expression until everything is written in terms of $\log x$, $\log y$, *and* $\log z$:

a. $\log x^2$ b. $\log(yz)^2$ c. $\log\sqrt[5]{x^3}$ d. $\log 100x^3$ e. $\log\left(\frac{xz^3}{x^{-1}y^2}\right)$

7. Answer the following questions about $f(x) = \log_3(x+4) - 2$.

a. What is $f(-1)$?

b. What is $f(-3)$?

c. What is the domain of $f(x)$?

e. What is the solution to $f(x) = -3$?

f. What is the solution to $f(x) = 1$

g. Find the inverse function $f^{-1}(x)$.

d. Find the zero of $f(x)$.

7.5. Find the inverse of $f(x) = 2 \cdot 4^x - 6$

8. Solve the following equations:

a. $\log_4 x - \log_4 2 = 2$

b. $2\log x - \log 3 = 2$

c. $\log_4 x - 2 = 1$

d. $4^x + 1 = 11$

e. $2\log x - 1 = \log 2x + 1$

f. $3 \cdot 2^{2x} + 3 = 27$

g. $\log_8(x-5) = \log_8(2x-9)$

h. $\log_5(3x+1) = 2$

i. $10^x + 4 = 16$

j. $\log_3 x + \log_3(x-2) = 1$

k. $\left(\frac{1}{4}\right)^{x-1} = 8$

l. $1 - 2\log_3 2x = -3$

m. $4\log_7 3x = 8$

n. $\log x + 2\log 2 = \log(2x+4) - 3\log 2$

o. $\log_x 8 = 3$

p. $\log 2 + 3\log(x-1) = 2$

q. $\frac{1}{2}\log(x+7) = \log(x+1)$

9. Solve the following word problems:

a. In 1985 Americans ate an average of 250 apples per year each. This number has fallen at a rate of decay of 1% per year. How many apples per person did Americans eat in 2002?

b. Based on problem (a) above, in what year was the annual per-person consumption equal to 225?

c. If the population of bacteria in a given culture rose from 50 to 150 in 22 hours then what was the hourly growth rate? (The *r* in $(1 + r)^x$)

d. An antique table is now worth $5000. If its value increased by a 9% annual rate of growth over the past 10 years, then what was its value 10 years ago?

e. How much will that table be worth seven years from now (assuming the rate of growth remains 9%)?

f. Money left is some bank grows at 5% per year. How many years will it take your money to double in value?

10. Given that $f(0) = 10$ *and* $f(4) = 20$, write the equation of $f(x)$ if it is a linear function. Then write its equation of it is an exponential function.

ANSWERS

1a. Dom: is all reals; Range is y>-1; asymptote is y=-1 b. Dom is reals; range is y<3; asymptote is y=3
c. domain is all reals; range is y<-1; asymptote is y=-2

2a. y-intercept is (0,4.5); zero is –2 b. y-intercept is (0,-2); zero is 4

3a. 2 b. 4 c. -3/2 d. 3/2 e. 2/3 f. 1 g. 1/3 h. 4 or –1 i. 3 or $\sqrt{3}$ ($-\sqrt{3}$ is extraneous)

4a. 3 b. -2 c. 2 d. -2 e. -1/2 f. 2 g. 3.5 h. 4/3

5*a.* $\log 45$ *b.* $\log\frac{9}{2}$ *c.* $\log_4 3$ *d.* $\log\frac{25}{32}$ *e.* $\log_7\frac{4x^2y}{9}$ *f.* $\log_3\frac{2}{y}$ *g.* $\log 72$ *h.* $\log_5\frac{12}{27} = \log_5\frac{4}{9}$

6*a.* $2\log x$ *b.* $2\log y + 2\log z$ *c.* $\frac{3}{5}\log x$ *d.* $\log 100 + 3\log x$ *e.* $2\log x + 3\log z - 2\log y$

7a. -1 b. -2 c. x>-4 d. 5 e. -11/3 f. 23 g. $y = 3^{x+2} - 4$ 7.5 $f^{-1}(x) = \log_4(\frac{x+6}{2})$

8*a*. 32 *b*. $\sqrt{300}$ *or* $10\sqrt{3}$ *c*. 64 *d*. 1.66 *e*. 200 *f*. $\frac{3}{2}$ *g*. *no solutions* *h*. 8

i. 1.08 *j*. 3 *k*. $-\frac{1}{2}$ *l*. $\frac{9}{2}$ *m*. $\frac{49}{3}$ *n*. $\frac{2}{15}$ *o*. 2 *p*. $\sqrt[3]{50}+1$ *or* 4.68 *q*. 2

9*a*. 211 *b*. 10.5 *years or* 1995 *c*. 5.1% *d*. \$2112 *e*. \$9140 *f*. 14.2 *years*

10. if linear then $f(x)=10+2.5x$; if exponential then $f(x)=10\cdot 1.189^{x}$