Working with Laws of Logarithms (Section 8.5) **May 26, 2009**
Do all work on separate paper.

1. For each part below, condense the expression below into a single logarithm and then evaluate it.

a. $\log_6 12 - \log_6 2$

b. $2\log_4 3 - 2\log_4 6$

c. $3\log 2 + \log 25 - \log 2$

d. $\log_9 12 - 2\log_9 2$

e. $\frac{1}{2}\log_5 16 - (\log_5 10 + \log_5 2)$

f. $\frac{1}{2}\log_6 3 + 0.5\log_6 12$

g. $2\log_{16} 6 - \left(2\log_{16} 3 + \frac{1}{2}\log_{16} 4\right)$

h. $\log_8 18 - 2(\log_8 24 - 2\log_8 2)$

2a. Is $2\log(3x) = \log(3x^2)$ *or* $\log(9x^2)$?? Try a few numbers with your calculator if you are not sure.

2b. Is $\log 4x^2 = 2\log 4x$? Why or why not? Try a few numbers with your calculator.

3. Condense the following into one logarithm:

a. $2\log x + 3\log 3$

b. $\frac{1}{3}\log 8 + \frac{2}{3}\log 27$

c. $2\log(xy) - \log(2x)$

d. $2\log 3x - (\log 9 + \log 6x)$

e. $2\log(9x) + \log x - 3\log(2x)$

f. $2\log x + \log(x-3) + \log 5$

g. $2\log(3x) - 3\log 2x + \log 4x$

4. Condense the logs on one side of the equation until to get something in the form $\log_a b = c$. Then rewrite as an exponential equation and solve it. Note: extraneous solutions occur when an answer would cause you to take the log of zero or a negative number in the original equation:

Example: $\log_2 x + \log_2(x+2) = 3$

$\log_2(x^2 + 2x) = 3$

$x^2 + 2x = 8$ *or* $x^2 + 2x - 8 = 0$

$(x-2)(x+4) = 0$ so x=2 or x=-4

-4 is extraneous because it would cause us to take the log of a negative number.

a. $\log 4 + \log x = 2$

b. $\log_4 x - \log_4 5 = 1$

c. $\log_8(2x^2) = 2$

d. $\log_6 x + \log_6(x+5) = 2$

e. $\frac{1}{2}\log_4(x-4) + \log_4 2 = 2$

f. $2\log_5(x-2) - \log_5 3 = 2$

5. Expand the following logs: Write in terms of $\log x, \log y,$ *and* $\log z$ (and log of a number):

Example: $\log 2x^2 y = \log 2 + \log x^2 + \log y = \log 2 + 2\log x + \log y$

a. $\log \frac{3}{x^3}$

b. $\log(2x^2 \cdot \sqrt{y})$

c. $\log\left(\frac{7}{\sqrt[3]{x}}\right)$

d. $\log\left(\frac{3\sqrt{y}}{5x^2}\right)$

e. $\log\sqrt{3xy}$

f. $\log\left(\frac{1}{x^2}\right)$

g. $\log\left(\sqrt[3]{x^2yz}\right)$

6. Solve the following equations using logarithms and a calculator. Remember the change-of-base formula ($\log_b x = \frac{\log x}{\log b}$):

a. $3^x - 7 = 29$

b. $2 \cdot 10^{x-1} = 80$

c. $4^x = 0.17$

ANSWERS

1a. 1 b. -1 c. 2 d. ½ e. -1 f. 1 g. 1/4 h. -1/3

2a. $\log(9x^2)$ b. no, it can be $\log(2x)^2 = 2\log 2x$ *or* $\log 4 + 2\log x$

3a. $\log 27x^2$ b. $\log 18$ c. $\log\left(\frac{xy^2}{2}\right)$ d. $\log\frac{x}{6}$ e. $\log\frac{81}{8}$ f. $\log(5x^2 - 15x)$

g. $\log\frac{9}{2}$

4a. 25 b. 20 c. $\sqrt{32}$ *or* $4\sqrt{2}$ (negative is extraneous) d. 4 (-9 is extran)

e. 68 f. $2 + 5\sqrt{3}$ ($2 - 5\sqrt{3}$ *is extraneous*)

5a. $\log 3 - 3\log x$ b. $\log 2 + 2\log x + 0.5\log y$ c. $\log 7 - \frac{1}{3}\log x$

d. $\log 3 + 0.5\log y - \log 5 - 2\log x$ e. $\frac{1}{2}\log x + \frac{1}{2}\log y + \frac{1}{2}\log 3$ f. $-2\log x$

g. $\frac{2}{3}\log x + \frac{1}{3}\log y + \frac{1}{3}\log z$

6a. 3.262 b. 2.602 c. -1.278