

## Logarithms

1. Make a table for the function  $f(x) = 2^x$ . Use at least  $-4, -3, -2, -1, 0, 1, 2, 3, 4$  for  $x$  values.
2. Graph that function on graph paper. Make sure that  $x$  and  $y$  are on the same scale. The scale will have to accommodate  $-2^4$  through  $2^4$ .
3. Now make another table representing the function  $f^{-1}(x)$  by making a table just like the other one but with the two columns switched (so for example  $f^{-1}(8) = 2$ ).
4. Graph  $f^{-1}$ . If all is correct, the graph of  $f^{-1}$  looks just like the graph of  $f$  reflected about the line  $y = x$ , like any inverse function.
5. The function  $f^{-1}$ , the inverse of the exponential function  $2^x$ , is called a *logarithm* function. In particular it is called the *logarithm base 2*. We write it as  $\log_2$ . So for example we may write  $\log_2(8) = 3$ , which is another way of saying  $2^3 = 8$ . Often we omit the function parentheses and write it  $\log_2 8 = 3$ . Fill in the table of exponential equations and their equivalent logarithm versions. (The symbol  $\Leftrightarrow$  means “is equivalent to.”)

$$2^3 = 8 \quad \Leftrightarrow \quad \log_2 8 = 3$$

$$2^4 = 16 \quad \Leftrightarrow$$

$$\Leftrightarrow \log_2 \frac{1}{8} = -3$$

$$2^0 = 1 \quad \Leftrightarrow$$

$$\Leftrightarrow \log_2 2 = 1$$

6. The above logarithms have a *base* of 2. This logarithm asks a question: What power of 2 yields the input to the function? For example, to find  $\log_2 16$  we ask, what power of 2 yields 16? What is that number?
7. There are other logarithms with other bases. The *common logarithm* has a base of 10. Sometimes we write common logarithms without an explicit base: What is the value of  $\log 1000 = \log_{10} 1000$ ?