1. Remember your exponent properties!

2. General form of exponential function:

$$f(x) = ab^{x-h} + k$$

   or

$$f(x) = a(1+r)^{x-h} + k$$

3. In the above equations:

   a. If $a > 0$ then the graph is strictly above the asymptote; if $a < 0$ then the graph is strictly below the asymptote. (Why?)

   b. b is the *growth factor*. Each time we add 1 to x, we multiply $f(x)$ by b. If $b < 1$ then we have *exponential decay*; if $b > 1$ then we have *exponential growth*. (What happens if $b = 1$?) (In our course, always $b > 0$.)

   c. r is the *growth rate*. If $r < 0$ then we have *exponential decay*; if $r > 0$ then we have *exponential growth*.

   d. Notice how $b = 1+r$ and that the previous two items basically say the same thing in different ways. b represents what to multiply $f(x)$ by to get $f(x+1)$, and r represents what to multiply $f(x)$ by to get *the difference between* $f(x)$ and $f(x+1)$, often thought of as a percentage change. For example, consider the case where $f(x+1)$ is always 20% less than $f(x)$. You can either write the function as $(1+(-0.2))^x$ (so $r = -0.2$) or 0.8^x (so $b = 1+(-0.2) = 0.8$).

   e. h just shifts the whole graph to the right by h units. (So if $h < 0$ the graph shifts left.) Because of an exponent property,

$$ab^{x-h} + k = (ab^{-h})b^x + k$$

   So shifting the graph to the right by h is equivalent to multiplying a by b^{-h}.

   f. k determines the *asymptote* which is the *line* $y = k$.

4. To sketch a graph:

   a. Graph the y-intercept (which is the point $(0, f(0))$).

   b. Lightly graph the *asymptote*, the line $y = k$.

   c. Determine the end behavior: if $b < 1$ then the graph approaches the asymptote as $x \to \infty$; if $b > 1$ then the graph approaches the asymptote as $x \to -\infty$. On the side that does not approach the asymptote, the graph goes up if $a > 0$ and goes down if $a < 0$.

   d. (Optional) Graph the point $(h, f(h))$ which is $(h, a+k)$.

   e. Start on the (left or right) side of the graph that approaches the asmptote, very slightly above the asmptote if $a > 0$; slightly below if $a < 0$. Draw a curve that gradually moves from near the asmptote through the point(s) and then grows away from the asymptote in the proper direction based on a.

5. To solve an equation involving exponentials, try to get the base on the right and left to be the same; if so the exponents must be the same. For example:

$$10^{1-x} = 100^2 \quad (1)$$
$$10^{1-x} = (10^2)^2 \quad (2)$$
$$10^{1-x} = 10^4 \quad (3)$$
$$1-x = 4 \quad (4)$$
$$-x = 4-1 \quad (5)$$
$$x = -3 \quad (6)$$

Check:

$$10^{1-(-3)} = 10^4 \quad (7)$$
$$= (10^2)^2 \quad (8)$$
$$= 100^2 \quad (9)$$