

Polynomial application problems

1. A rectangular prism's length exceeds its width by 5 cm. Its height is 2 cm less than its width. Its volume is 70 cm^3 . What is its width?
2. A pyramid can be formed using equal-size balls. For example, 3 balls can be arranged in a triangle, then a fourth ball placed in the middle on top of them.

The function $p(n) = \frac{1}{6}n(n+1)(n+2)$ gives the number of balls in a pyramid, where n is the number of balls on each side of the bottom layer. (For the pyramid described above, $n = 2$.)

- a. Evaluate $p(2)$, $p(3)$, and $p(4)$.
 - b. If you had 1000 balls available and you wanted to make the largest possible pyramid using them, what would be the size of the bottom triangle, and how many balls would you use to make the pyramid? (This is almost impossible to solve algebraically; use your graphing calculator by finding a polynomial function you can set to zero, then "solve" by graphing that function (you will have to adjust the window size) and then using $\boxed{2^{\text{nd}}}$ $\boxed{\text{CALC}}$ zero to find the zero of that function (remember to move the "left bound" and "right bound" to opposite sides of the zero.)
3. A company manufactures cardboard boxes in the following way: they begin with 12"-by-18" pieces of cardboard, cut an x "-by- x " square from each of the four corners, then fold up the four flaps to make an open-top box.
 - a. Sketch a picture or pictures of the manufacturing process described above. Label all segments in your diagram with their lengths (these will be formulas in terms of x).
 - b. What are the length, width, and height of the box, in terms of x ?
 - c. Write a function $V(x)$ expressing the volume of the box.
 - d. Only some values of x would be meaningful in this problem. What is the interval of appropriate x -values?
 - e. Using the interval you just named, make the graph $V(x)$ on your calculator, then sketch it on paper.
 - f. Approximately what value of x would produce a box with maximum volume? (Use your calculator.)
 - g. What are the dimensions and the volume for the box of maximum volume?
 4. Here are two important formulas for a sphere of radius r :
 - The surface area of the sphere: $S(r) = 4\pi r^2$.
 - The volume of the sphere: $V(r) = \frac{4}{3}\pi r^3$.
 - a. The radius of the Earth is 3963 miles. Assuming the Earth is a perfect sphere (almost but not quite true), calculate its surface area and its volume.
 - b. An air balloon is shaped like a sphere and has a volume of 1000 cubic inches. What is the surface area of the balloon?
 - c. An astronomer discovers Planet X and Planet Y. Planet Y has twice the radius of Planet X. How do the planets' surface areas compare? How do their volumes compare?

more problems on back...

Name _____

5. Grain is often stored in a silo that is shaped like a cylinder topped by a hemisphere dome.

The volume of a silo is given by the formula $\frac{2}{3}\pi r^3 + \pi r^2 h$, where r is the radius of the cylinder and h is the height of the cylinder. (The first term is the volume of the hemisphere and the second term is the volume of the cylinder.)

- a. A farmer wants to design a silo whose cylindrical part has a height of 20 feet. The radius r has not yet been decided. Write a formula for function $V(r)$ giving the volume as a function of the radius.
- b. Draw a graph of function $V(r)$, using an appropriate interval of r -values as the domain.
- c. If the radius is chosen to be 6 feet, what will be the volume of the silo?
- d. The farmer wants the silo to have a volume between 3000 cubic feet and 3500 cubic feet. What are the possible values for the radius? (Use your calculator.)