

This is a guide to the stuff you should know for our upcoming test.

The test is on Chapter 8, Polynomials. The first section of this guide lists the things in Chapter 8 that you can safely ignore. The rest highlights the important topics to understand. However, understanding is only part of studying. The other big part is practicing. Do or redo as many homework problems as you can. You can also try other problems in the book, but remember that some of them require techniques we haven't learned.

Most (maybe all) of the test will be without calculators.

Summary of what *not* to study

Study all of Chapter 6 *except*:

- Anything involving the word “synthetic”
- Section 6.4: the difference of two cubes and special cubic factoring patterns
- Section 6.5: The Remainder Theorem
- All of Section 6.6
- Finite differences
- regression

Exponent properties (Book Section 6.1)

- $a^m a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $(ab)^m = a^m b^m$
- $a^{-m} = \frac{1}{a^m}$ (for $a \neq 0$)
- $a^0 = 1$ (for $a \neq 0$)
- $\frac{a^m}{a^n} = a^{m-n}$ (for $a \neq 0$)
- $(\frac{a}{b})^m = \frac{a^m}{b^m}$ (for $a \neq 0$)

Polynomial functions

(Book section 6.2)

- *Polynomial function*: A function that can be written in the form

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

(also called *standard form*).

- Each $a_k x^k$ is a *term*.
- a_n is the *leading coefficient*.
- n is the *degree*.
- a_0 is the *constant term*.
- Certain kinds of polynomials have their own names, based on degree:

| Degree | Type | Standard form |
|--------|-----------|---|
| 0 | Constant | a_0 |
| 1 | Linear | $a_1 x^1 + a_0$ |
| 2 | Quadratic | $a_2 x^2 + a_1 x^1 + a_0$ |
| 3 | Cubic | $a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0$ |
| 5 | Quartic | $a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x^1 + a_0$ |

- *Evaluating* a polynomial function is finding its *value* given a specific input value for the variable (usually x).

Graphing

- End behavior:
 - * Leading coefficient positive: As $x \rightarrow \infty$, $f(x) \rightarrow \infty$ (right end goes up);
 - * Leading coefficient negative: As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$ (right end goes down)
 - * Degree even: left end goes in same vertical direction as right end
 - * Degree odd: left end goes in opposite vertical direction from right end.
 - Example: if leading term is $-3x^5$ then as $x \rightarrow \infty$, $f(x) \rightarrow -\infty$ (right end goes down) because the coefficient is negative. As $x \rightarrow -\infty$, $f(x) \rightarrow \infty$ (left end goes up) (opposite of down) because degree 5 is odd.
- Factor to find zeros
 - Multiplicity 1: linear zero
 - Multiplicity even: “bump” (like a quadratic)
 - Multiplicity odd and greater than 1: *Inflection point* or “flex” or “twist”.

- To graph a specific point, pick x and compute $f(x)$ to find y . For example, to graph the y -intercept, plot $(0, f(0))$.
- To analyze a graph, use the factor theorem to determine the factors. Determine the multiplicity of the zeros to determine the powers of the factors. This gives you a formula like

$$a(x - 4)^3(x - 2)(x - (-1))^2$$

To determine a , just pick one point on the graph, (x_0, y_0) , and plug in like this:

$$a(x_0 - 4)^3(x_0 - 2)(x_0 - (-1))^2 = y_0$$

Then you can easily solve for a .

Polynomial arithmetic

(Book section 6.3)

- Adding and subtracting: just add/subtract like terms.

- Multiplying: Use the distributive law! Use the box:

| | | |
|------|--------|-------|
| · | $2x$ | $+3$ |
| x | $2x^2$ | $3x$ |
| -5 | $-10x$ | -15 |

Factoring and solving polynomial equations

(Book section 6.4)

- Don't worry about sum and difference of cubes
- Factoring by grouping: If the left and right half of standard form polynomial are divisible by the same polynomial, you can factor out that polynomial. You can use the distributive law box for this. Put the polynomial inside the box (left hand side on top, right hands side on bottom) and factor.

For example: Given $x^3 - 2x^2 - 9x + 18$ put it inside the box:

| | | |
|---|-------|---------|
| · | | |
| | x^3 | $-2x^2$ |
| | $-9x$ | 18 |

Then find the factors by filling in the outside.

| | | |
|-------|-------|---------|
| · | x | -2 |
| x^2 | x^3 | $-2x^2$ |
| -9 | $-9x$ | 18 |

- Factoring using quadratic form: In this case, you notice that the function is a quadratic with the right variable substitution. So you substitute, then use all you know about quadratics, then you "un-substitute." For example, to factor

$$x^4 + 3x^2 + 2$$

substitute u for x^2 . Then you need to factor

$$u^2 + 3u + 2$$

Seen as a quadratic, this is easy:

$$(u + 2)(u + 1)$$

Then “un-substitute.”

$$(x^2 + 2)(x^2 + 1)$$

Hopefully the resulting smaller-degree polynomials are easier to deal with. In this case you can factor $x^2 + 2$ using the Difference of Squares method (because $x^2 + 2 = x^2 - (\sqrt{-2})^2$). This yields $(x - i\sqrt{2})(x + i\sqrt{2})$, and you can factor $x^2 + 1$ similarly. Or you could use the Quadratic Formula.

- *Division method.* This method works great when you have a way to find at least one factor. You can divide by that factor (typically with long division) and then have a lower-degree polynomial to factor.

If you have a *zero* k , by the factor theorem, you also have a *factor* $x - k$, so you can use the same method.

Remember that if you have a complex zero, its complex conjugate is also a zero, and it's usually much easier to divide by the product of both corresponding factors. For example if you know that $2i$ is a zero, you know that $-2i$ is also a zero, so both $x - 2i$ and $x + 2i$ are factors. So you divide by the product $(x - 2i)(x + 2i)$ which is $x^2 + 4$. This avoids spending too much time with complex numbers.

How do you find zeros or factors? The easiest method is to be told. Another way is to use your graphing calculator to find a zero. A calculator is necessarily approximate, but if it looks like a nice round number, you can divide by the corresponding factor and see if it's right.

By the way, once you get the degree down to 2 (quadratic) you can always use the quadratic equation if you can't factor or use the Difference of Squares method.

Remainder and Factor Theorems

(Book section 6.5)

We can skip the remainder theorem.

The *Factor Theorem* tells us that each zero k corresponds to the factor $x - k$ and each factor $x - k$ corresponds to the zero k .

Rational Zeros

(Book section 6.6)

Skip this section!

Fundamental Theorem of Algebra

(Book section 6.7)

Fundamental Theorem of Algebra: Any polynomial of degree $n > 0$ has at least one complex (maybe real) root. (A *root* is a zero.)

Since you can divide that polynomial by the factor corresponding to that root, and get a polynomial of degree $n - 1$, and since that polynomial must also have a root as long as it is of degree greater than zero, the Fundamental Theorem really means that a polynomial of degree n has n roots. (We count repeated roots as many times as they are repeated. So, for example, we say that $(x - 2)^3$ has 3 roots: 2, 2 and 2. I know this sounds strange.)

Polynomial Graphing

(Book section 6.8)

Don't worry about "turning points."

Finite differences are really cool and related to calculus in an interesting way, but don't worry about those either.

Modeling

(Book section 6.9)

Example 1 is good but skip the rest.