## Factor Theorem

1. Thinking, for the moment, about numbers, what are all the factors of 12 (not counting 1 and 12)?

2. What are the factors of $2x^2 + 2x - 4$?

3. What are the zeros of $2x^2 + 2x - 4$?

4. Considering only the factors of the form $x - k$, what is the relationship between the $k$ values and the zeros?

5. Make a conjecture (educated guess) for the *Factor Theorem*: A polynomial $f(x)$ has a factor $x - k$ if and only if...

6. Explain why the "if" direction is true, i.e. why it's true that *if* $f(x)$ has a factor $x - k$, *then* ... . (The "only if" direction is harder.)

7. Now let's use the theorem to factor a polynomial. Use your calculator to find the zeros of $x^3 - 3x^2 - x + 3$ (graph and find the zeros by eye or use 2nd CALC `zero`).

8. Once you have found the zeros, what are some factors? (In this case, these are all the factors.)

## Remainder Theorem

9. What if you divide by a polynomial that's not a factor? If $f$ is the polynomial $x^2 + 7x - 5$, What are the quotient and remainder of $f(x) \div (x - 2)$? (Use long division.)

10. Notice the 2 in our divisor, $x - 2$. What is $f(2)$?

11. Make a conjecture for the *Remainder Theorem*: If a polynomial $f(x)$ is divided by $x - k$, the remainder is...

Answers:

1. 2, 3, 4, 6
2. 2, $x-1$, $x+2$, $2x-2$, $2x+4$
3. 1, $-2$
4. The zeros are the $k$ values.
5. *Factor Theorem*: A polynomial $f$ has a factor $x-k$ if and only if $k$ is a zero of $f$ (i.e. if and only if $f(k)=0$).
6. If $x-k$ is a factor of $f$, that means $f(x)=(x-k)q(x)$ for some polynomial $q$. So $f(k)=(k-k)q(k)=0q(k)=0$.
7. Zeros are $-1$, 1 and 3.
8. Factors are $x+1$, $x-1$, and $x-3$.
9. quotient: $x+9$; remainder: 13
10. $f(2)=13$.
11. *Remainder Theorem*: If a polynomial $f(x)$ is divided by $x-k$, the remainder is $f(k)$.