Algebra 2 Name:
Grouping / Quadratic Form Date:

Polynomials:
Grouping and the Quadratic Form
(See Sec 6.4 of the book for more info.)

How can we factor $10x^3 + 20x^2 + x + 2$?

Grouping –

The pattern for factoring by grouping looks like this:

Examples:

$3x^3 - 6x^2 + x - 2$

$x^3 + 3x^2 + 10x + 30 = 0$

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Does this polynomial look like anything we've seen before? $x^4 + 3x^2 + 2$

Quadratic Form –
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The pattern for the quadratic form looks like this:

Examples:

$4x^4 - 5x^2 - 9$ $x^4 + 10x^2 + 16 = 0$

Homework:

1. Factor the following with grouping. Make sure your answer is **fully factored**.

a. $x^3 - 3x^2 + 5x - 15$ b. $2x^3 - 5x^2 - 4x + 10$

c. $x^3 + x^2 - x - 1$ d. $x^3 + 2x^2 - 9x - 18$

2. Factor the following using some combination of factoring out a common factor, grouping, and difference of squares.

a. $-2x^4 + 6x^3 + 8x^2 - 24x$ b. $x^5 + 5x^4 - x - 5$

3. Solve the following equations by factoring. Find all **real and imaginary solutions**. Also, whenever factoring, make sure one side is zero!

a. $x^3 + 2x^2 - 9x = 18$

b. $x^3 - 2x^2 + 4x = x + 6$

c. $\frac{-11}{17}(x^4 - 1)(x^2 + 2x + 1) = 0$

d. $2x^4 + 5x^3 - 14x^2 = 35x$

4. Factor the following with quadratic form. Make sure everything is fully factored.

a. $x^4 - x^2 - 6$

b. $x^4 - 7x^2 + 6$

c. $x^4 - 16$

d. $x^4 + 6x^2 - 40$

5. Factor the following using some combination of monomial factors, grouping, and quadratic form.

a. $x^5 - 3x^3 - 4x$

b. $-2x^5 - 4x^3 + 6x$

6. Find all solutions to the following equations. This means real and imaginary/complex ones.

a. $x^3 + 7x^2 + 5x + 34 = x + 6$

b. $\frac{x^4 - 11x^2}{2} + 14 = 2$

c. $15 + 5x - 3x^2 - x^3 = 0$

d. $x^4 = 9$

7a. Write a polynomial *expression* of degree 3.

7b. Write a polynomial *equation* of degree 4.

7c. Write a polynomial *function* of degree 2.

Answers:

1a. $(x^2+5)(x-3)$ b. $(x^2-2)(2x-5)$ c. $(x+1)(x-1)(x+1)$

d. $(x+3)(x-3)(x+2)$

2a. $-2x(x^3-3x^2-4x+12) = -2x(x-3)(x+2)(x-2)$

b. $(x^4-1)(x+5) = (x-1)(x+1)(x^2+1)(x+5)$

3a. $(x^2-9)(x+2)=0$ *so* $x=3,-3,-2$

b. $(x^2+3)(x-2)=0$ *so* $x=\pm i\sqrt{3},2$

c. $(x^2-1)(x^2+1)(x+1)^2=0$ *so* $x=1,-1,i,-i$

d. $x(x^2-7)(2x+5)=0$ $x=0,-2.5,\pm\sqrt{7}$

4a. $(x^2-3)(x^2+2)$ b. $(x^2-1)(x^2-6) = (x+1)(x-1)(x^2-6)$

c. $(x^2-4)(x^2+4) = (x-2)(x+2)(x^2+4)$ d. $(x^2-4)(x^2+10) = (x+2)(x-2)(x^2+10)$

5a. $x(x^2+1)(x-2)(x+2)$ b. $-2x(x^4+2x^2-3) = -2x(x^2+3)(x+1)(x-1)$

6a. -7, $\pm 2i$ b. $\pm\sqrt{3}, \pm 2\sqrt{2}$ c. -3, $\pm\sqrt{5}$ d. $\pm\sqrt{3}, \pm i\sqrt{3}$

7a. (Examples). x^3+2x^2-3x+4 7b. $4x^4+3x^3=2$ 7c. $f(x)=x^2-4$