

Solving systems by elimination – Day 2

Yesterday we began using the *elimination method* for solving a system of two equations. The key step in the method is to add or subtract the two equations to eliminate one of the variables. Later in the assignment we began to see problems where it was necessary to do a multiplication in one of the equations before adding or subtracting.

Here's the new idea for today: Sometimes the easiest way to eliminate a variable involves using **multiplication in both equations**. An example is shown below. In the example, one of the equations has a $3y$ and the other has a $2y$. Think of a *common multiple* of $3y$ and $2y$: the easiest one to use is $6y$. Now choose the multiplications that will make both equations have a $6y$. Thus the equation with the $3y$ should be multiplied by 2, and the equation with the $2y$ should be multiplied by 3.

EXAMPLE 1 Using Multiplication First

$$\begin{array}{rcl} \text{Solve the linear system.} & 4x - 3y = 11 & \text{Equation 1} \\ & 3x + 2y = -13 & \text{Equation 2} \end{array}$$

SOLUTION

The equations are arranged with like terms in columns. You can get the coefficients of y to be opposites by multiplying the first equation by 2 and the second equation by 3.

$$\begin{array}{rcl} 4x - 3y = 11 & \text{Multiply by 2.} & 8x - 6y = 22 \\ 3x + 2y = -13 & \text{Multiply by 3.} & \underline{9x + 6y = -39} \\ & & 17x = -17 \quad \text{Add the equations.} \\ & & x = -1 \quad \text{Solve for } x. \end{array}$$

Substitute -1 for x in the second equation and solve for y .

$$\begin{array}{rcl} 3x + 2y = -13 & \text{Write Equation 2.} & \\ 3(-1) + 2y = -13 & \text{Substitute } -1 \text{ for } x. & \\ -3 + 2y = -13 & \text{Simplify.} & \\ y = -5 & \text{Solve for } y. & \end{array}$$

The solution is $(-1, -5)$.

Outline of the elimination method using multiplication first

1. Write both equations in the standard form $Ax + By = C$, which lines up the like terms.
2. Pick either x or y to eliminate. Think of a common multiple for the terms having that variable. Multiply in one or both equations to get two equations that have that common multiple.
3. If the equations have opposite terms (such as $4x$ and $-4x$), add the equations.
OR: If the equations have identical terms (such as $-4x$ and $-4x$), subtract the equations.
4. One of the variables should have been eliminated. Solve for the remaining variable.
5. Substitute the value you've found into one of the original equations. Solve for the other variable.
6. Write the solution. Check your answer.

You try it

Directions: Solve these systems using the elimination method. You will need to use multiplication first.

1. $2x + 3y = 11$
 $5x + 4y = 17$

Hint: Use multiplications to get $10x$ in both equations.

2. $5x + 4y = 22$
 $7x - 6y = -4$

Hint: $12y$ is a common multiple of $4y$ and $6y$.

3. $3x - 2y = 16$
 $2x + 5y = -2$

4. Solve by elimination. Start by choosing multiples so that the x variable is eliminated.

$$\begin{aligned}6x + 10y &= 2 \\ 10x - 20y &= 40\end{aligned}$$

5. Solve by elimination. Start by choosing multiples so that the y variable is eliminated.

$$\begin{aligned}6x + 10y &= 2 \\ 10x - 20y &= 40\end{aligned}$$

4 and 5 were the same system solved two different ways. You should have gotten the same answer. If you didn't, find your mistake.

6. Solve using the elimination method. Something different happens in this problem.

$$4x + 6y = 1$$

$$6x + 9y = 3$$

More problems — answer on separate paper

Directions: On separate paper, solve each system using the elimination method.

Hints:

- For most problems you will need to multiply before adding or subtracting.
- In problems **13–18** the equations need to be rearranged so that like terms are lined up. For example, in problem **13**, subtract y from both sides in the first equation, and add $6x$ to both sides in the second equation. After that, proceed as usual.

7. $x + 2y = -3$

$$x - 4y = 15$$

10. $4x - 3y = -3$

$$4x + 5y = 5$$

13. $4x = -11 + y$

$$y = -6x - 9$$

16. $4x = 5y - 14$

$$3y - 8x = -14$$

8. $-x - 5y = 30$

$$2x - 7y = 25$$

11. $4x + 5y = -2$

$$5x - 4y = -23$$

14. $x = 2y - 3$

$$2y = 3x + 13$$

17. $5x = 4y - 30$

$$2x + 3y = -12$$

9. $-x + 8y = 16$

$$3x + 4y = 36$$

12. $9x - 4y = -18$

$$-3x + 8y = 6$$

15. $4y = 15 - 3x$

$$2y = 3x + 21$$

18. $\frac{2}{3}y = 10 + 4x$

$$5x = \frac{1}{3}y - 8$$