Solving systems by elimination

Here is another method for solving a system of two equations. Sometimes this method is easier than either the graphing method or the substitution method. The idea of this new method is that sometimes, by adding or subtracting the two equations, you can ***eliminate*** one of the variables. After that, it's easy to solve for the other variable.

Example A: Solve this system: $x - y = 25;\ x + y = 71$

	$x - y = 25$
	$\underline{x + y = 71}$
Add the equations (the y's are eliminated):	$2x = 96$
Solve for x:	$x = 48$
Now pick either of your original equations:	$x - y = 25$
Put in the value found for x:	$48 - y = 25$
Solve for y:	$48 = 25 + y$
	$23 = y$
Answer:	$(x, y) = (48, 23)$

In general, here are the steps of this method:

1. Write both equations in the standard form $Ax + By = C$, which lines up the like terms.

2. If the equations have opposite terms (such as $4x$ and $-4x$), add the equations.
OR: If the equations have identical terms (such as $-4x$ and $-4x$), subtract the equations.

3. One of the variables should have been eliminated. Solve for the remaining variable.

4. Substitute the value you've found into one of the original equations. Solve for the other variable.

5. Write the solution. Check your answer.

Example B: Solve this system: $3y = 24 - 6x;\ 4x = 18 - 3y$

Put equations in std. form to line up like terms:	$6x + 3y = 24$
	$\underline{4x + 3y = 18}$
<u>Subtract</u> top equation minus bottom equation:	$2x = 6$
Solve for x:	$x = 3$
Now pick either of your original equations:	$6x + 3y = 24$
Put in the value found for x:	$6 \cdot 3 + 3y = 24$
Simplify, and solve for y:	$18 + 3y = 24$
	$3y = 6$
	$y = 2$
Answer:	$(x, y) = (3, 2)$

Example C: Solve this system: $2x + 5y = 19;\ 3y = 2x + 5$

Put equations in std. form to line up like terms:	$2x + 5y = 19$
	$\underline{-2x + 3y = 5}$
<u>Add</u> the equations:	$8y = 24$
Solve for y:	$y = 3$
Now pick either of your original equations:	$2x + 5y = 19$
Put in the value found for y:	$2x + 5 \cdot 3 = 19$
Simplify, and solve for x:	$2x + 15 = 19$
	$2x = 4$
	$x = 2$
Answer:	$(x, y) = (2, 3)$

Problems (elimination method)

Directions: Solve these systems using the ***elimination method*** as explained on page 1.

1. $2x - 3y = 12$
$4x + 3y = 42.$

2. $-2x + 6y = -26$
$8x + 6y = -16.$

3. $-2x + 4y = 10$
$-2x + y = 1.$

4. $2y - x = 8$
$x - 5y = -11$

Hint: First you need to line up like terms.

5. $5y = x + 17$
$x - 2y = -11$

Hint: In 1st equation, must move x to the left side.

6. $3x = 5y + 2$
$y = 3x - 10.$

7. Solve this problem by setting up a system of equations then using the elimination method.

A convenience store sells coffee in small and large sizes.
One customer buys 2 small coffees and 3 large coffees for \$5.20.
Another customer buys 2 small coffees and 1 large coffee for \$2.80.
What are the prices for a small coffee and for a large coffee?

8. Solve this problem by setting up a system of equations then using the elimination method.

Donna and Emily are sisters. Donna's age and Emily's age add up to 31.
Donna is 3 years younger than Emily. How old are the two sisters?

Multiplying before you add or subtract

In the elimination method, a variable is eliminated when you add or subtract only if there existed matching terms (with opposite or same signs). Here is a problem without matching terms, so adding or subtracting wouldn't work.

Example:

$$\begin{aligned} 2x + y &= 8 \\ 4x + 3y &= 18 \end{aligned}$$

There is a method for dealing with this kind of situation. You can multiply one of the equations by a number of your choice (in every term) in order to create matching terms. In the example, the bottom equation has a $3y$, and if the top equation is multiplied by 3 then it will have a $3y$ too. After that, you can subtract and finish solving in the usual way.

$2x + y = 8$ ***multiplying by 3→*** $6x + 3y = 24$

$4x + 3y = 18$ ***keeping the same→*** $\underline{4x + 3y = 18}$

now subtract: $2x = 6$

Finish solving as shown in Example B on page 1.

Problems (elimination method with multiplying)

9.
$$\begin{aligned} x + 3y &= 10 \\ 4x + 2y &= 20 \end{aligned}$$

Hint: Get a $4x$ in the top equation also.

10.
$$\begin{aligned} 3x + 2y &= 7 \\ 5x - y &= -10 \end{aligned}$$

11. $-2x + 3y = 5$
$x - 6y = 2$

12. $4x - y = 39$
$2x + 5y = 25$

13. $x + y = 12$
$2x + 3y = 31$

14. $4x + 3y = 38$
$x - y = 6$

Unusual outcomes from the elimination method

Usually when we're doing the elimination method, at the adding or subtracting step, one of the two variables drops out and we're left with one variable so we can solve the equation. However, there are two other outcomes that sometimes occur.

Situation: Both variables drop out, and you're left with a false equation.

Watch what happens when we subtract the equations in this example:

$$\begin{array}{r} 2x + 3y = 9 \\ \underline{2x + 3y = 5} \\ 0 \;+ 0 \;= 4 \end{array}$$

Both of the variables dropped out, and we're left with a **false equation** (0 + 0 doesn't equal 4). Whenever this happens, it means that the problem **has no solutions.**

Situation: Both variables drop out, and you're left with a true equation.

Watch what happens when we add the equations in this example:

$$\begin{array}{r} 4x - 5y = -7 \\ \underline{-4x + 5y = 7} \\ 0 \;+ 0 \;= 0 \end{array}$$

Both of the variables dropped out, and we're left with a **true equation** (0 + 0 does equal 0). Whenever this happens, it means that the problem **has infinitely many solutions.**

Problems (watch for unusual outcomes)

15. Solve: $2x - 3y = 8$
$-2x + 3y = -8$

16. Solve: $-5x + y = 7$
$-5x + y = 12$

17. Solve: $x - 4y = 5$
$-x + 4y = 5$